



Beam, beam, Beam

——无衍射光束的产生与调控

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Outline

- Helmholtz Equation
- Gaussian beam

- Bessel beam

Nondiffracting beams

- Airy beam

- Mathieu and Weber beam

- Plasmonic counterpart



Helmholtz Equation

电磁波波动方程

$$\nabla^2 \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

代入时谐电磁场

$$\vec{E}(\vec{x}, t) = \vec{E}(\vec{x}) \exp(-i\omega t)$$

$$\vec{B}(\vec{x}, t) = \vec{B}(\vec{x}) \exp(-i\omega t)$$

可得 赫姆霍兹方程：

$$\nabla^2 u + k^2 u = 0 \quad \text{最简单的解是: } \vec{E}(\vec{x}) = \vec{E}_0 e^{i\vec{k} \cdot \vec{x}}$$

平面电磁波

平面波传播波形不改变，但它只是**wave**，不是**beam**



(I) Gaussian beam

亥姆霍兹方程的波束解

考虑波束能量分布具有轴对称性，中间场强最大，靠近边缘强度迅速衰减。在横截面上具有这种分布性质的最简单函数就是**高斯函数**：

$$e^{-\frac{x^2+y^2}{w^2}}$$

参数 w 表示光束的宽度
波束宽度通常还是 z 的函数

波幅通常也是 z 的函数，我们以 $u(x,y,z)$ 代表电磁场任意直角分量，它可以具有如下形式

$$u(x, y, z) = g(z)e^{-f(z)(x^2+y^2)}e^{ikz}$$



$$u(x, y, z) = g(z)e^{-f(z)(x^2+y^2)}e^{ikz}$$

式中 e^{ikz} 表示以来与 z 的主要因子，剩下的因子中还有对 z 的缓变函数 $g(z)$ 和 $f(z)$ ，

因子 $e^{-f(z)(x^2+y^2)}$ 是限制波束的空间宽度的因子。

因子 $g(z)$ 主要表示波的振幅，同时也含有传播因子中与纯平面因子 e^{ikz} 偏离的部分。令

$$\phi(x, y, z) = g(z)e^{-f(z)(x^2+y^2)}$$

它满足 z 的缓变振幅近似。因此它对 z 的展开式中的高次项可以忽略。



根据亥姆霍兹方程 $\nabla^2 u + k^2 u = 0$

将 $u(x, y, z) = \phi(x, y, z)e^{ikz}$ 代入
得到：

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \left(\cancel{\frac{\partial^2 \phi}{\partial z^2}} + 2ik \frac{\partial \phi}{\partial z} - \cancel{k^2 \phi} \right) + \cancel{k^2 \phi} = 0$$

忽略对z求导的高次项得到

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + 2ik \frac{\partial \phi}{\partial z} = 0$$

傍轴波动方程



将 ϕ 的试探解形式代入

$$(x^2 + y^2)[2gf^2 - ikgf'] - [2fg - ikg'] = 0$$

上式要对任意 x, y 都成立，必须两括号内都为零

$$\begin{cases} 2f^2 = ikf' & (1) \end{cases}$$

$$\begin{cases} 2fg = ikg' & (2) \end{cases}$$

上式（1）的解形式为

$$f(z) = \frac{1}{A + \frac{2i}{k}z}$$



对比 (1) (2) 式, 可以看出 $g(z)=f(z) \times \text{常数}$

$$g(z) = \frac{u_0}{1 + \frac{2i}{kA} z}$$

将 $f(z)$ 变换, 得到

$$f(z) = \frac{1}{A(1 + \frac{4z^2}{k^2 A^2})} (A - \frac{2i}{k} z)$$

令 $A = w_0^2$

$$w^2(z) = A(1 + \frac{4z^2}{k^2 A^2}) = w_0^2 [1 + (\frac{2z}{kw_0^2})^2]$$



则：

$$f(z) = \frac{1}{w^2(z)} \left(1 - \frac{2iz}{kw_0^2}\right)$$

那么高斯函数变为

$$e^{-f(z)(x^2+y^2)} = \exp\left[-\frac{x^2+y^2}{w^2(z)} \left(1 - \frac{2iz}{kw_0^2}\right)\right]$$

同样 $g(z)$ 可以写为

$$g(z) = \frac{u_0}{\sqrt{1 + \left(\frac{2z}{kw_0^2}\right)^2}} e^{-i\varphi} = u_0 \frac{w_0}{w} e^{-i\varphi}$$

其中

$$\varphi = \arctan\left(\frac{2z}{kw_0^2}\right)$$



最终得到光束场强函数

$$u(x, y, z) = u_0 \frac{w_0}{w} e^{-\frac{x^2 + y^2}{w^2}} e^{-i\Phi}$$

其中

$$\Phi = kz + \frac{k(x^2 + y^2)}{2z \left[1 + \left(\frac{kw_0^2}{2z} \right)^2 \right]} - \varphi$$

$$w^2(z) = w_0^2 \left[1 + \left(\frac{2z}{kw_0^2} \right)^2 \right]$$

高斯光束的传播特性

$$u(x, y, z) = u_0 \frac{w_0}{w} e^{-\frac{x^2 + y^2}{w^2}} e^{-i\Phi}$$

振幅

相因子

$$e^{-\frac{x^2 + y^2}{w^2}}$$

限制波束宽度

$$w^2(z) = w_0^2 \left[1 + \left(\frac{2z}{kw_0^2} \right)^2 \right]$$

波束宽度由 $w(z)$ 代表，在 $z=0$ 处波束具有最小宽度，称为**光束腰部——束腰**。

$$u_0 \frac{w_0}{w}$$

z 轴上的振幅， u_0 是束腰位置的振幅。



$$\Phi = kz + \frac{k(x^2 + y^2)}{2z \left[1 + \left(\frac{kw_0^2}{2z} \right)^2 \right]} - \varphi$$

光束波阵面是等相位面，由相位 Φ =常数确定。

当 $z=0$ 时， $\Phi=0$ ，说明 $z=0$ 平面是一个波阵面，即光束腰部波阵面与 z 轴垂直。

当远离束腰部位 $z \gg kw_0^2$ $z + \frac{x^2 + y^2}{2z} = \text{常数}$

小量展开 $\left(1 + \frac{x^2 + y^2}{z^2} \right)^{\frac{1}{2}} \approx 1 + \frac{x^2 + y^2}{2z^2}$

得到： $r = (x^2 + y^2 + z^2)^{\frac{1}{2}} \approx \text{常数}$

波阵面为一球面



再由 $z^2 \gg x^2 + y^2$

$$\left(1 + \frac{x^2 + y^2}{z^2}\right)^{\frac{1}{2}} \approx 1 + \frac{x^2 + y^2}{2z^2}$$

所以等相位面方程可写为:

$$z\left(1 + \frac{x^2 + y^2}{z^2}\right)^{\frac{1}{2}} \approx \text{常数}$$

即 $r = (x^2 + y^2 + z^2)^{\frac{1}{2}} \approx \text{常数}$

因此，在远处波阵面变为以束腰中心为球心的一个球面。总的波阵面就是从腰部的平面逐渐过渡到远处的球面形状。



另外，在远处 $z \gg kw_0^2$

$$w(z) = w_0^2 \left[1 + \left(\frac{2z}{kw_0^2} \right)^2 \right] \approx \frac{2z}{kw_0^2}$$

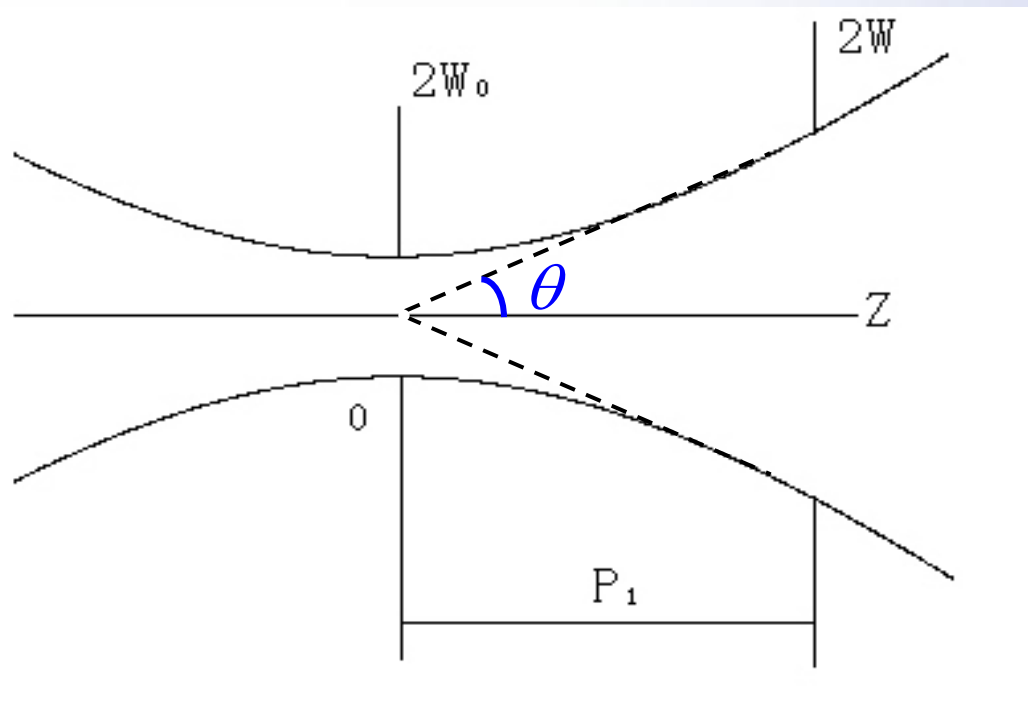
波束发散角由 $\tan \theta = w/z$ 确定，所以

$$\tan \theta \approx \frac{2}{kw_0^2}$$

对应一给定波长的电磁波，当 w_0 愈小时，发散角愈大。因此，要求良好的聚焦效果，发散角必须足够大；如果要求良好定向，则光束宽度不能太小。

$$\Delta k_{\perp} \cdot w_0 \approx O(1)$$

高斯光束示意图



光束在传播中有发散，明显的衍射效应

Q1: 是否存在不衍射的光束?



(II) Bessel beam

Helmholtz Equation $\nabla^2 u + k^2 u = 0$

在柱坐标下变为

$$\frac{1}{r} \frac{\partial u}{\partial r} \left(\frac{1}{r} \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} + k^2 u = 0$$

令 $u(r, \theta, z) = v(r, \theta)Z(z)$, 代入方程:

$$Z \left[\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial v}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + k^2 v \right] = -v \frac{d^2 Z}{dz^2}$$

继续作分离变量, 分离 r 和 θ :



作分离变量，引入常数 λ ，得到：

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial v}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + (k^2 - \lambda)v = 0$$

$$\frac{d^2 Z}{dz^2} + \lambda Z = 0$$

接着令 $v(r, \theta) = R(r)\Phi(\theta)$, 代入得到：

$$\frac{r^2}{R} \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{dR}{dr} \right) + (k^2 - \lambda)R \right] = - \frac{1}{\Phi} \frac{d^2 \Phi}{d\theta^2}$$



最终得到三个分离变量微分方程：

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dR}{dr} \right) + \left(k^2 - \lambda - \frac{\mu}{r^2} \right) R = 0$$

$$\frac{d^2 \Phi}{d\theta^2} + \mu \Phi = 0$$

$$\frac{d^2 Z}{dz^2} + \lambda Z = 0$$

$$\Phi_m(\theta) = \begin{cases} \cos(m\theta) \\ \sin(m\theta) \end{cases} \text{ 或 } \begin{cases} \exp(im\theta) \\ \exp(-im\theta) \end{cases}$$

$$Z_\lambda(z) = \begin{cases} \cos(k_z z) \\ \sin(k_z z) \end{cases} \text{ 或 } \begin{cases} \exp(ik_z z) \\ \exp(-ik_z z) \end{cases}$$

这里 $\mu = m^2$, $\lambda = k_z^2$ 。

令 $k_r^2 = k^2 - k_z^2 = k^2 - \lambda$ 。此时方程 (1) 变为

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dR}{dr} \right) + \left(k_r^2 - \frac{m^2}{r^2} \right) R = 0$$

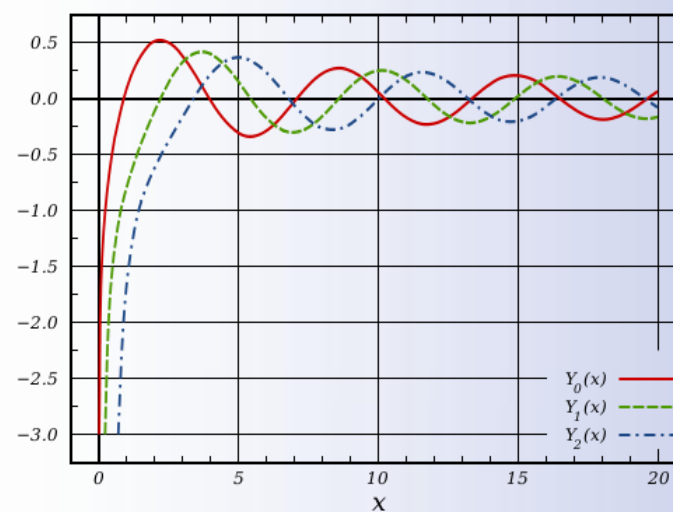
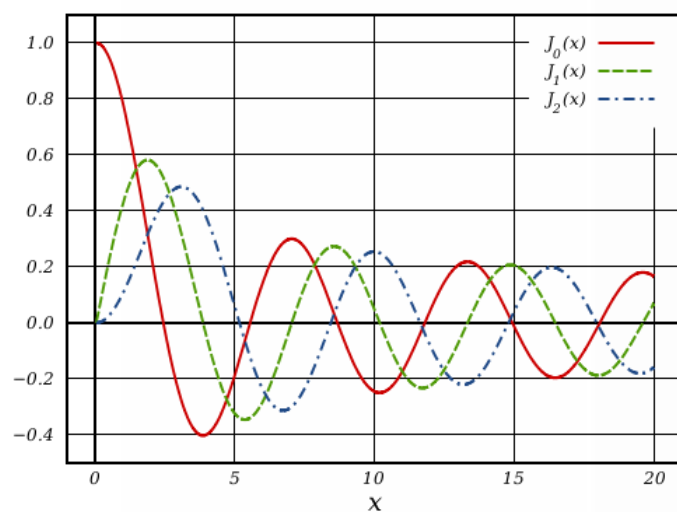
Bessel 方程

因此我们得到径向函数的通解

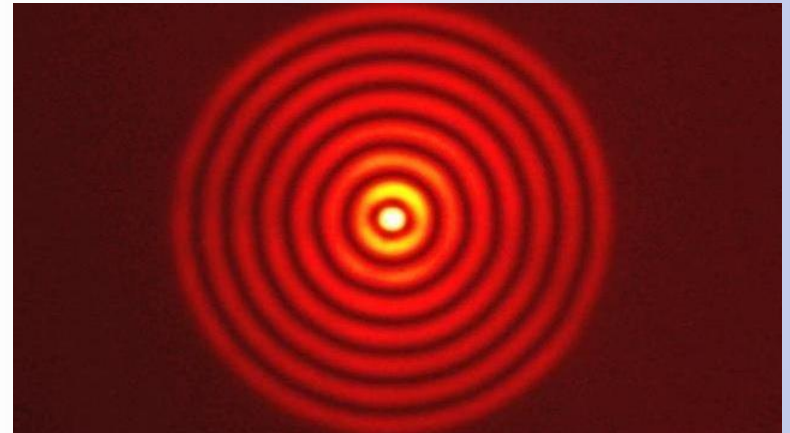
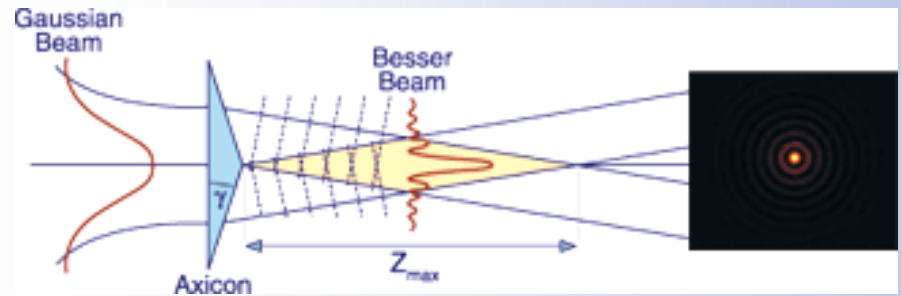
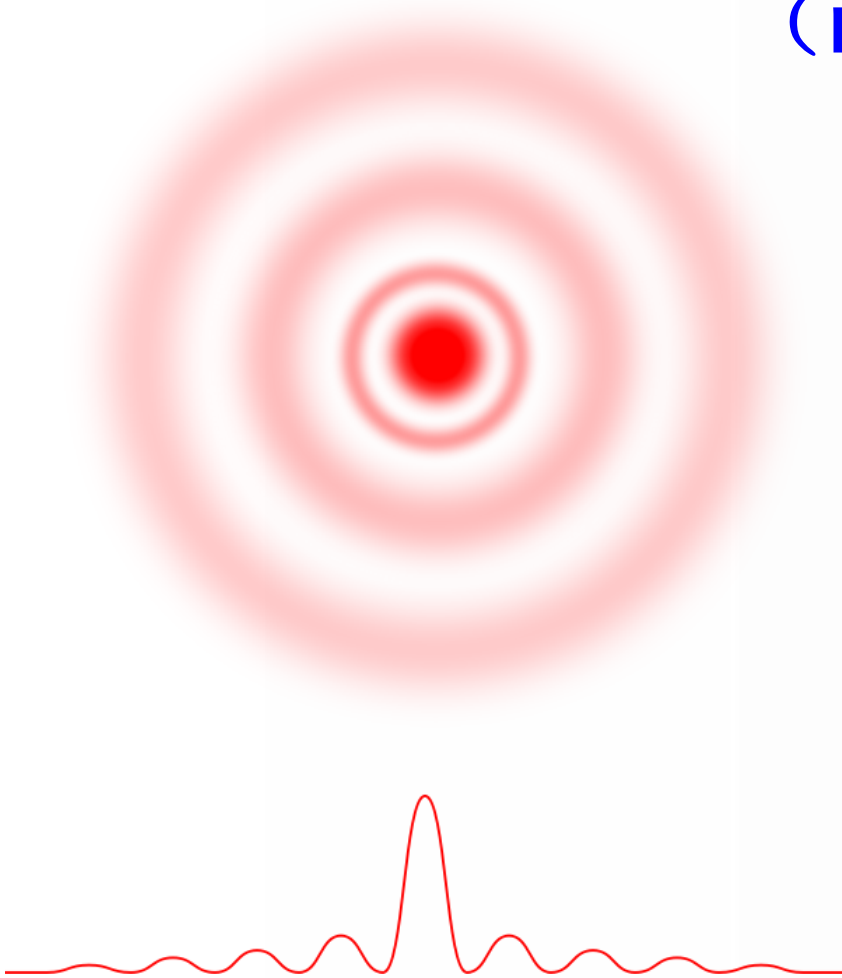
$$R(r) = CJ_m(k_r r) + DN_m(k_r r)$$

由边界 $R(0)$ 有限值，得到 $D=0$ 。

所以最终的径向解就是Bessel函数 $J_m(kr)$ 形式，
而旋转切向则为振荡解，振荡波矢 m 决定 J 的阶数！



零阶Bessel beam ($m=0$)





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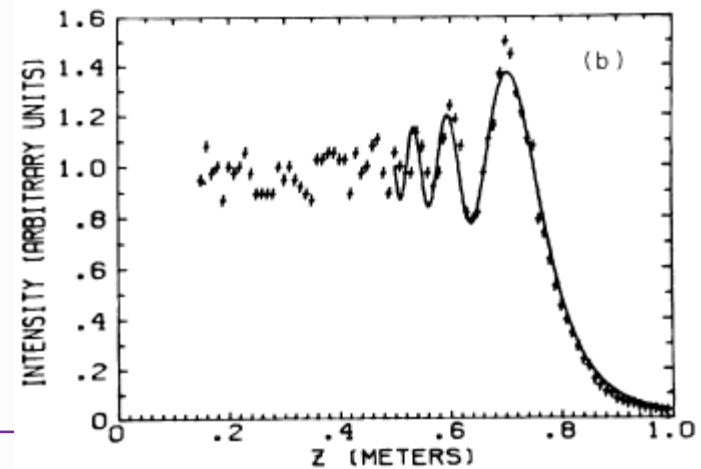
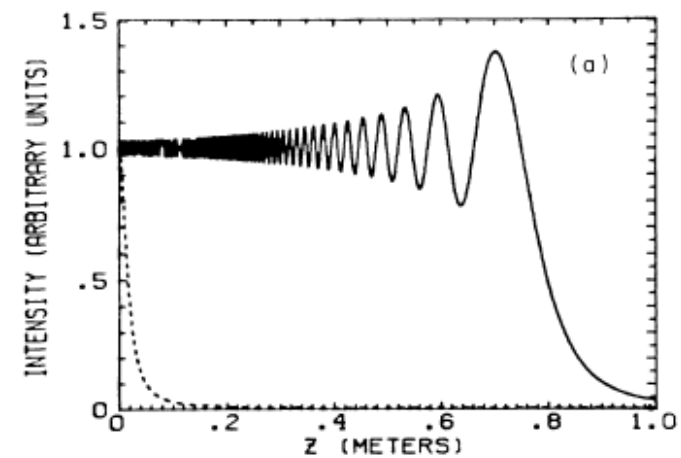
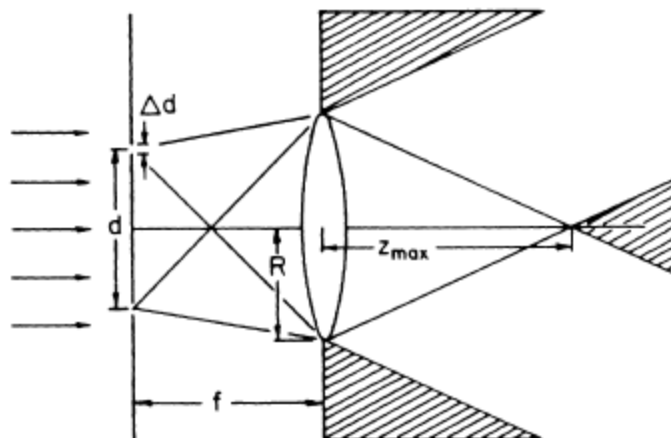
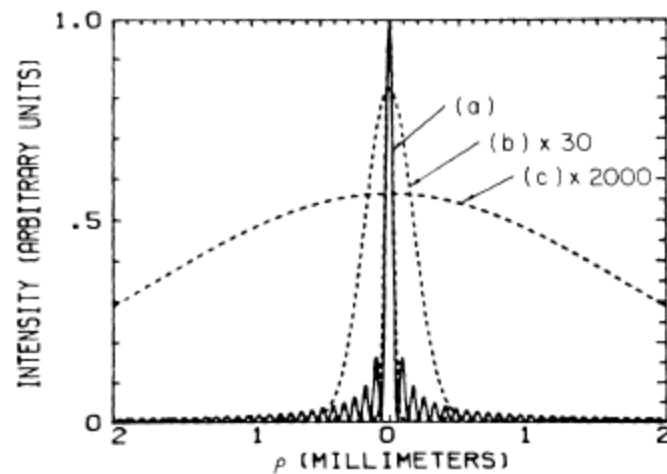
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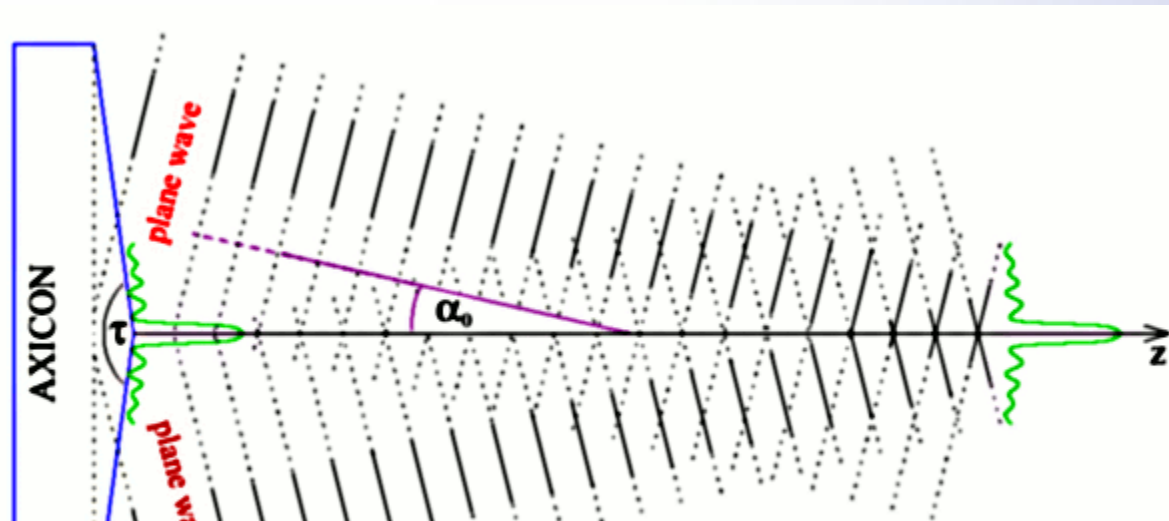
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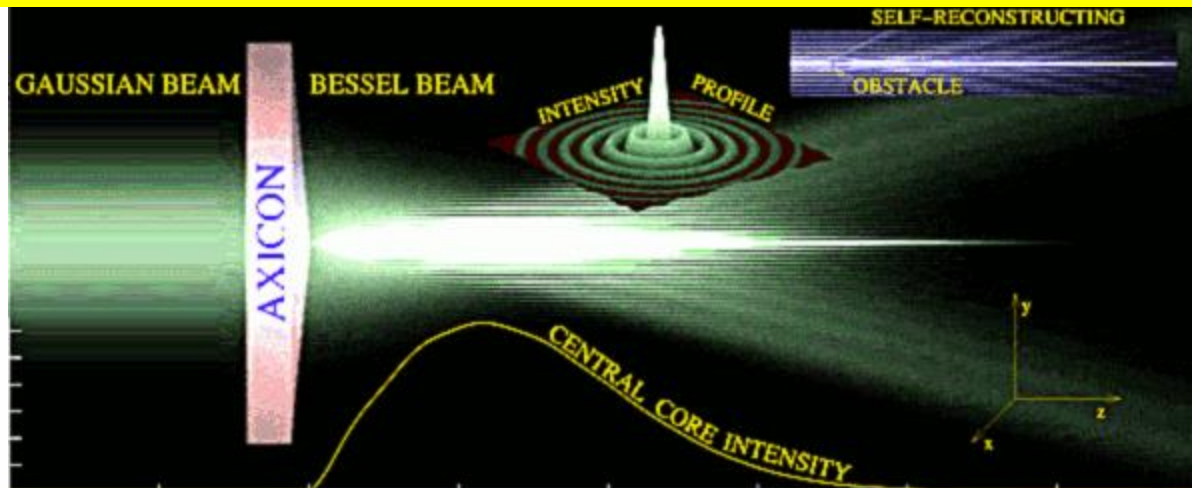
Diffraction-Free Beams

J. Durnin and J. J. Miceli, Jr.

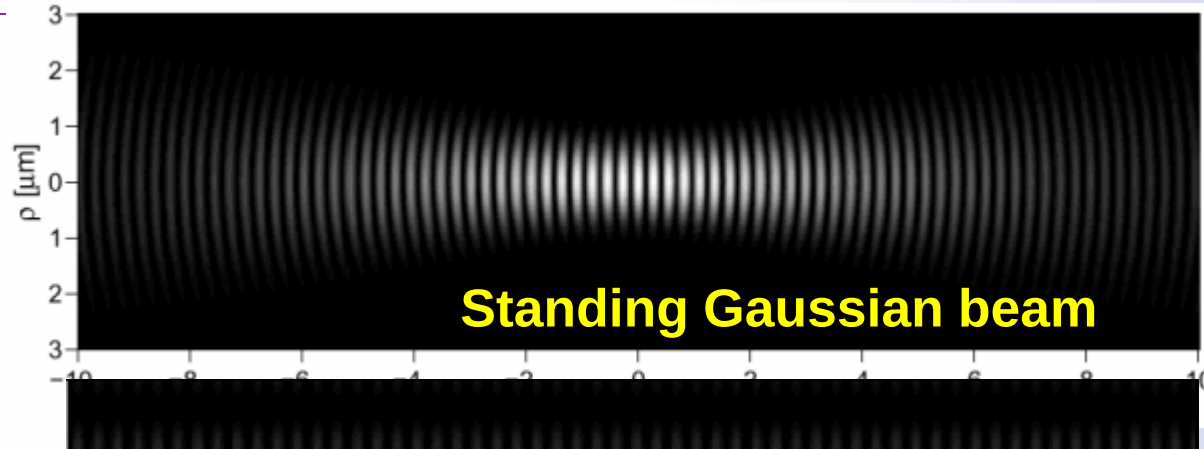
The Institute of Optics, University of Rochester, Rochester, New York 14627



Q2: 是否存在弯曲的 nondiffracting beam?



<http://www.isibrno.cz/omitec/index.php?action=bessintr.html>

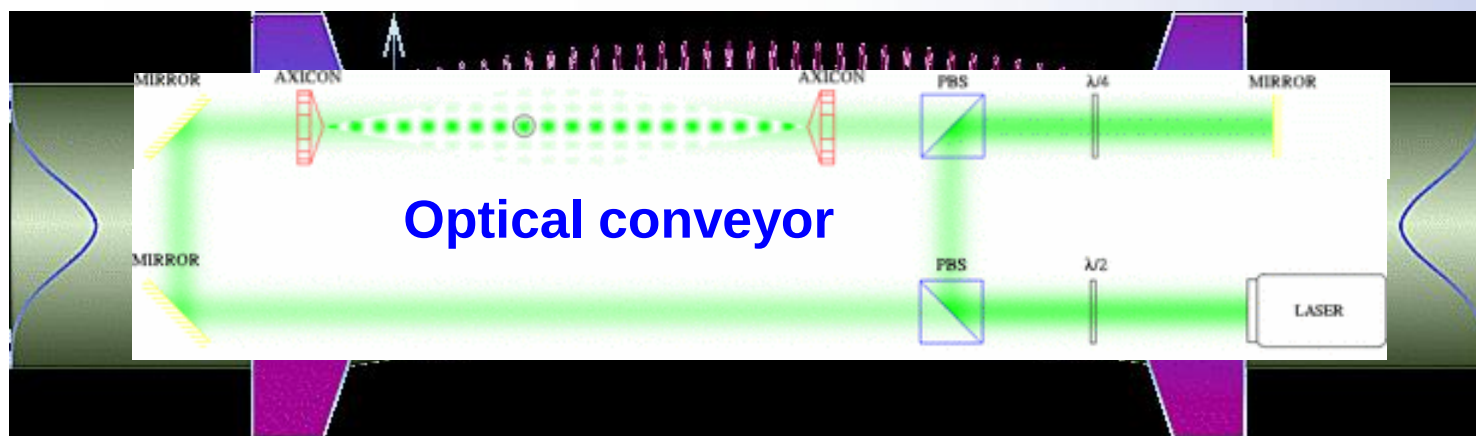


Standing Gaussian beam

Q2: 是否存在弯曲的 nondiffracting beam?



Standing Bessel beam





(III) Airy beam

- 回到Cartesian傍轴波动方程

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + 2ik \frac{\partial \phi}{\partial z} = 0$$

- 如果考虑场在y方向均匀，即无波动，则方程变为二维傍轴方程，如下

$$\frac{\partial^2 \phi}{\partial x^2} + 2ik \frac{\partial \phi}{\partial z} = 0$$

- 对照薛定谔方程

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = i\hbar \frac{\partial \psi}{\partial t}$$

形式一致！



Airy 函数

Airy方程: $y'' - xy = 0$

$y = \text{Ai}(x)$ 是方程的一支解。

$$\text{Ai}(x) = \frac{1}{\pi} \int_0^{\infty} \cos\left(\frac{t^3}{3} + xt\right) dt$$

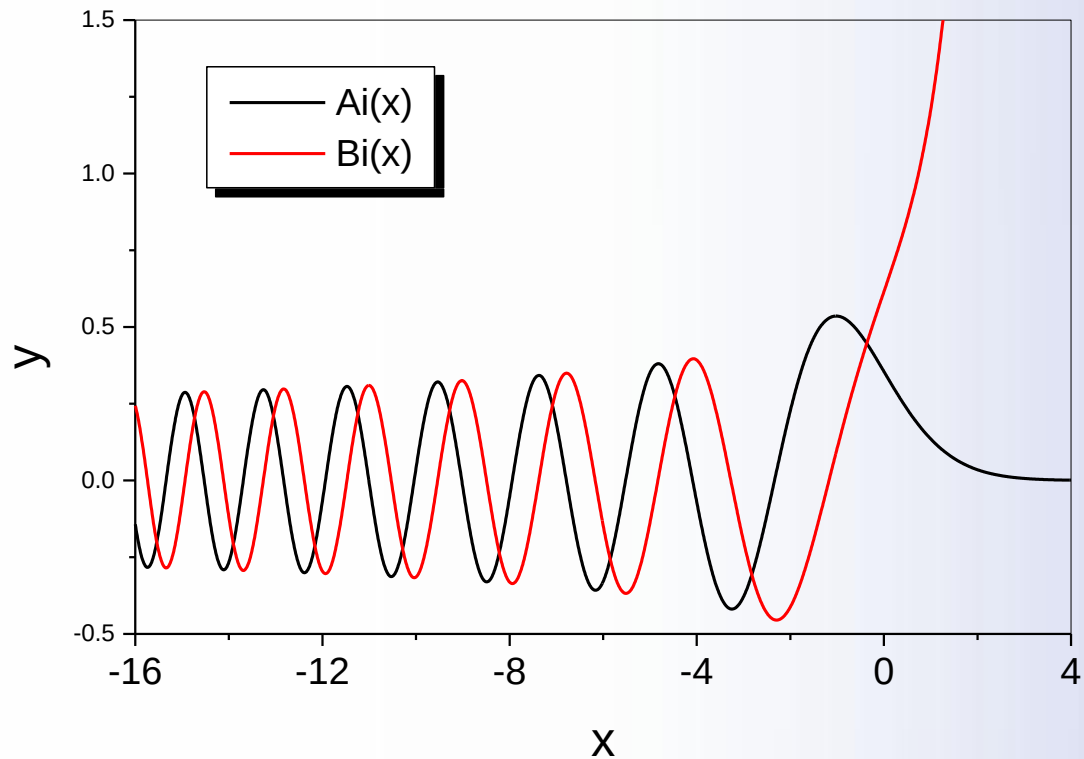
还有一支

$$\text{Bi}(x) = \frac{1}{\pi} \int_0^{\infty} \left[\exp\left(-\frac{t^3}{3} + xt\right) + \sin\left(\frac{t^3}{3} + xt\right) \right] dt$$

通解 :

$$y = C_1 \text{Ai}(x) + C_2 \text{Bi}(x)$$

$Ai(x)$ $Bi(x)$ 函数图象





Ai(x)函数具体形式

$$\text{Airy方程: } \frac{d^2 Ai(x)}{dx^2} - x Ai(x) = 0$$

对其做变量变换

$$\xi = x^{3/2}, w(\xi) = Ai(x) / \sqrt{x}$$

得到：

$$\frac{d^2 w}{d\xi^2} + \frac{1}{\xi} \frac{dw}{d\xi} - \left(\frac{1}{3}\right)^2 \left(4 + \frac{1}{\xi^2}\right) w = 0$$

虚宗量贝塞尔方程

$$Ai(x) = \begin{cases} \frac{1}{\pi} \sqrt{\frac{x}{3}} K_{1/3} \left(\frac{2}{3} x^{\frac{3}{2}} \right), & (x > 0) \\ \frac{\sqrt{-x}}{3} \left[J_{1/3} \left(\frac{2}{3} (-x)^{\frac{3}{2}} \right) + J_{-1/3} \left(\frac{2}{3} (-x)^{\frac{3}{2}} \right) \right], & (x < 0) \end{cases}$$

贝塞尔函数的马甲

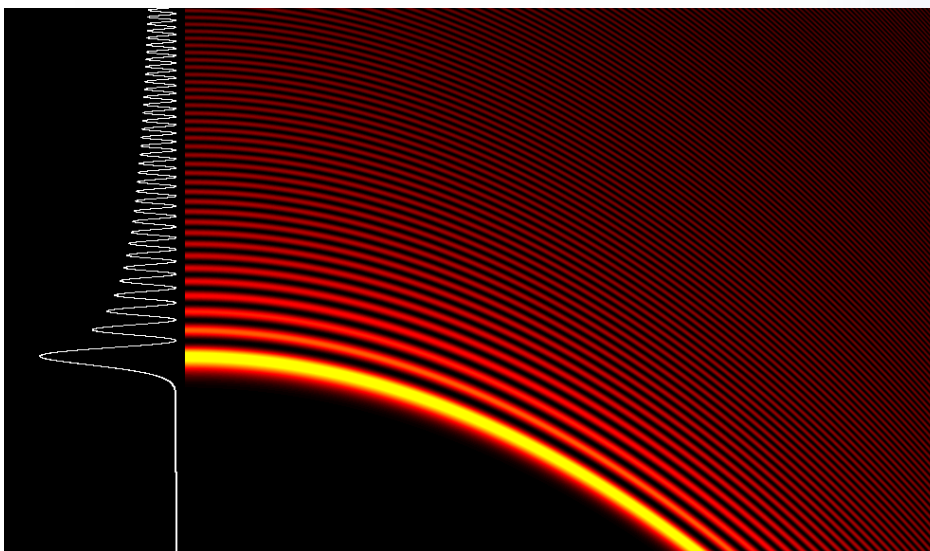
Airy function

根据Ai(x)求薛定谔方程：

$$\frac{\partial^2 \phi}{\partial x^2} + 2ik \frac{\partial \phi}{\partial z} = 0$$

通过一系列步骤，我们得到无衍射严格解：

$$\phi(x, z) = \text{Ai}\left(ax - \frac{a^4 z^2}{4k^2}\right) \exp\left(i \frac{a^6 z^3}{12k^3} - i \frac{a^3 xz}{2k}\right)$$



Airy beam

不衍射，不发散，
自弯曲，自加速，
自修复



如何根据Ai(x)求薛定谔方程：

$$\frac{\partial^2 \phi}{\partial x^2} + 2ik \frac{\partial \phi}{\partial z} = 0$$

不失一般性令 $\phi = \text{Ai}(f_1) \exp(f_2)$ ，其中 f_1 和 f_2 是 x, z 的函数。

然后求出 $\frac{\partial \phi}{\partial z}$ 、 $\frac{\partial^2 \phi}{\partial x^2}$ 的具体形式，代入波动方程，并利用：

$$\text{Ai}''(x) = x \text{Ai}(x)$$

最后对比Ai(x)和Ai'(x)前面的系数，可得：

$$\begin{cases} \frac{\partial^2 f_1}{\partial x^2} + 2 \frac{\partial f_1}{\partial x} \frac{\partial f_2}{\partial x} = -2ik \frac{\partial f_1}{\partial z} \\ \left(\frac{\partial f_2}{\partial x} \right)^2 + \frac{\partial^2 f_2}{\partial x^2} + f_1 \left(\frac{\partial f_1}{\partial x} \right)^2 = -2ik \frac{\partial f_2}{\partial z} \end{cases}$$



考虑无衍射形式的解，这要求 $f_1 = ax + f_3(z)$ ，其中 f_3 仅是 z 的函数。代入上述方程组，然后分离变量，并考虑初值条件，最后可解得：

$$f_3(z) = -\frac{a^4}{4k^2} z^2,$$

$$f_1(x, z) = ax - \frac{a^4 z^2}{4k^2},$$

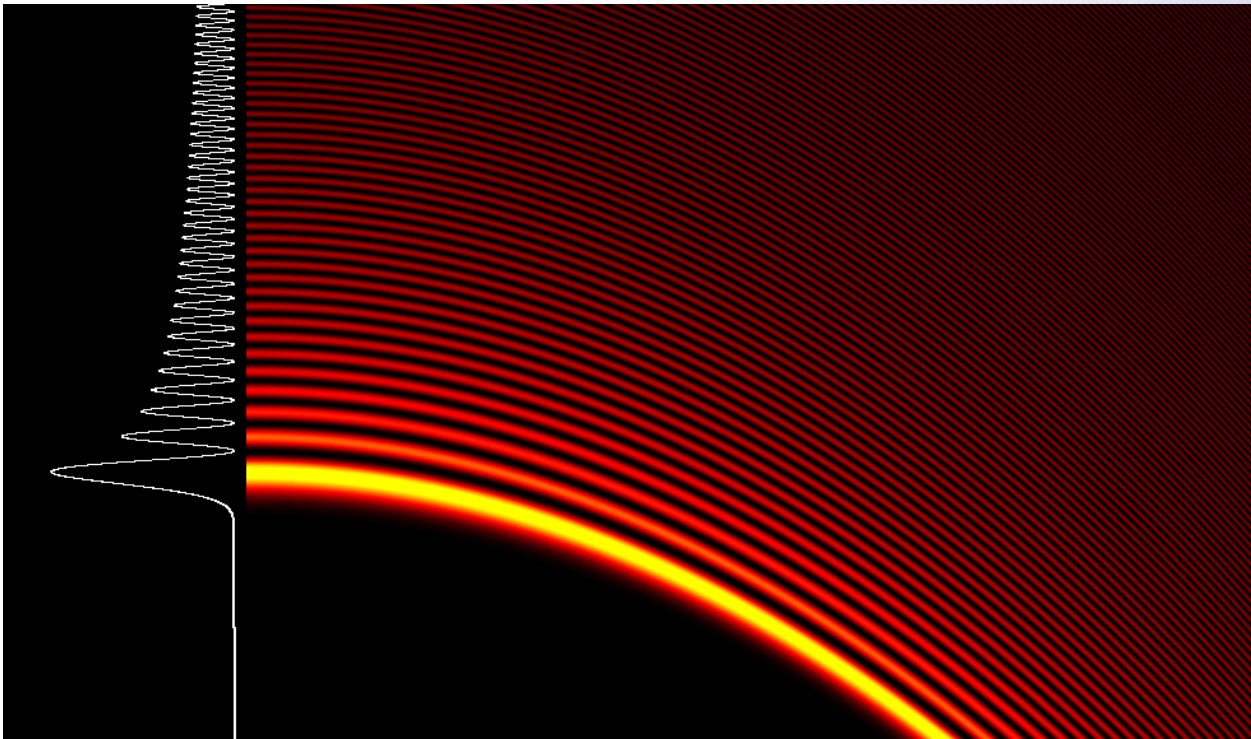
$$f_2(x, z) = i \frac{a^6 z^3}{12k^3} - i \frac{a^3 xz}{2k}.$$

将 f_1 和 f_2 代入 ϕ 的表达式即可得到最后结果。

$\phi(x, 0) = \text{Ai}(ax)$ 初值条件下得到非衍射严格解：

$$\phi(x, z) = \text{Ai}\left(ax - \frac{a^4 z^2}{4k^2}\right) \exp\left(i \frac{a^6 z^3}{12k^3} - i \frac{a^3 xz}{2k}\right)$$

Airy beam 图象



不衍射，不发散，
自弯曲，自加速，自修复



Nonspreading wave packets

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(Received 30 June 1978; accepted 12 September 1978)

$$\psi(x,0) \approx \frac{1}{\sqrt{\pi}} \left(\frac{\hbar^{2/3}}{-Bx} \right)^{1/4} \sin[\pi/4 + 2(-Bx)^{3/2}/3\hbar]$$

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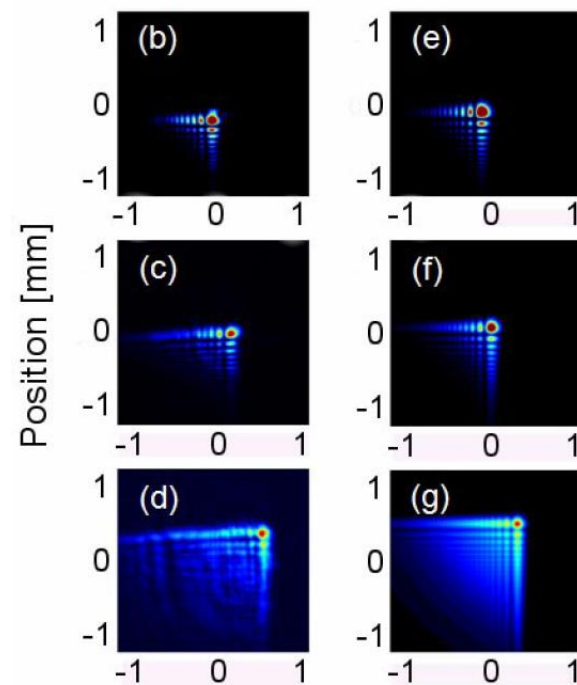
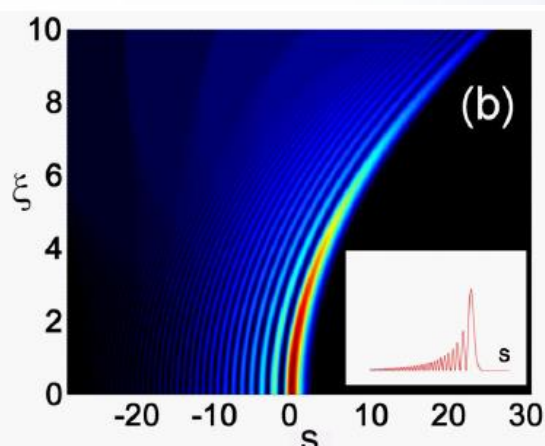
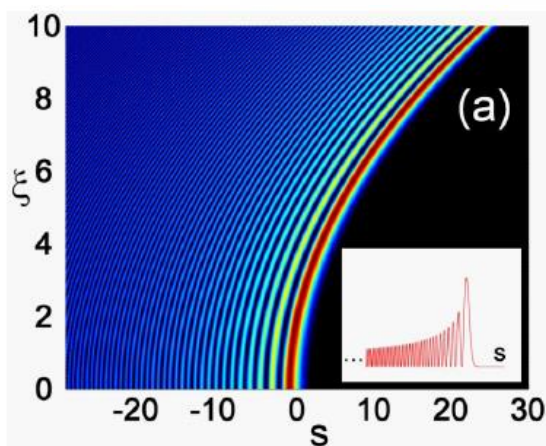


Observation of Accelerating Airy Beams

G. A. Siviloglou, J. Broky, A. Dogariu, and D. N. Christou

College of Optics/CREOL, University of Central Florida, Orlando

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Infinite $\rightarrow$ finite

$$i \frac{\partial \phi}{\partial \xi} + \frac{1}{2} \frac{\partial^2 \phi}{\partial s^2} = 0,$$

$$\phi(\xi, s) = \text{Ai}(s - (\xi/2)^2) \exp(i(s\xi/2) - i(\xi^3/12)).$$

有限能量

$$\phi(0, s) = \text{Ai}(s) \exp(as)$$

$$\phi(\xi, s) = \text{Ai}(s - (\xi/2)^2 + ia\xi) \exp(as - (a\xi^2/2) - i(\xi^3/12) + i(a^2\xi/2) + i(s\xi/2)).$$

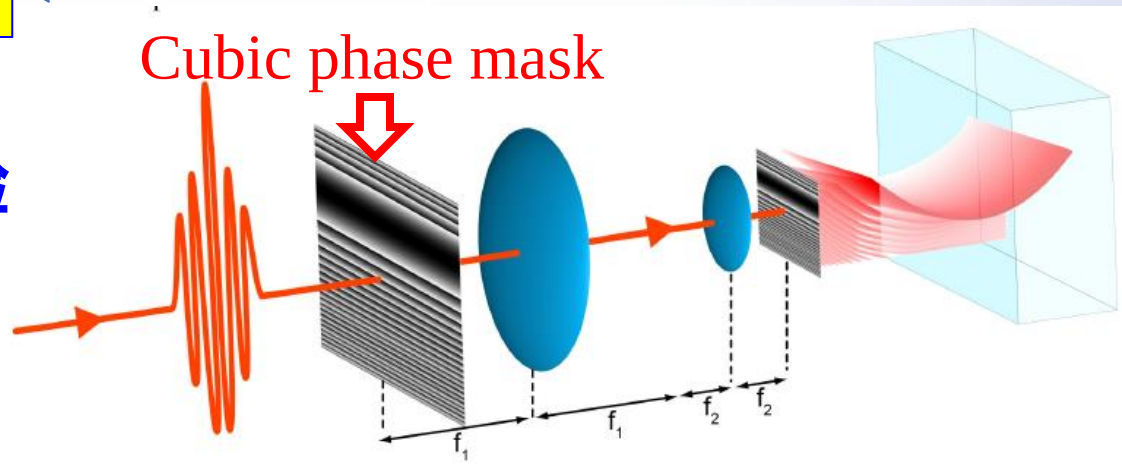
Fourier Transformation

$$\Phi(k) \propto \exp(-ak^2) \exp(ik^3/3)$$

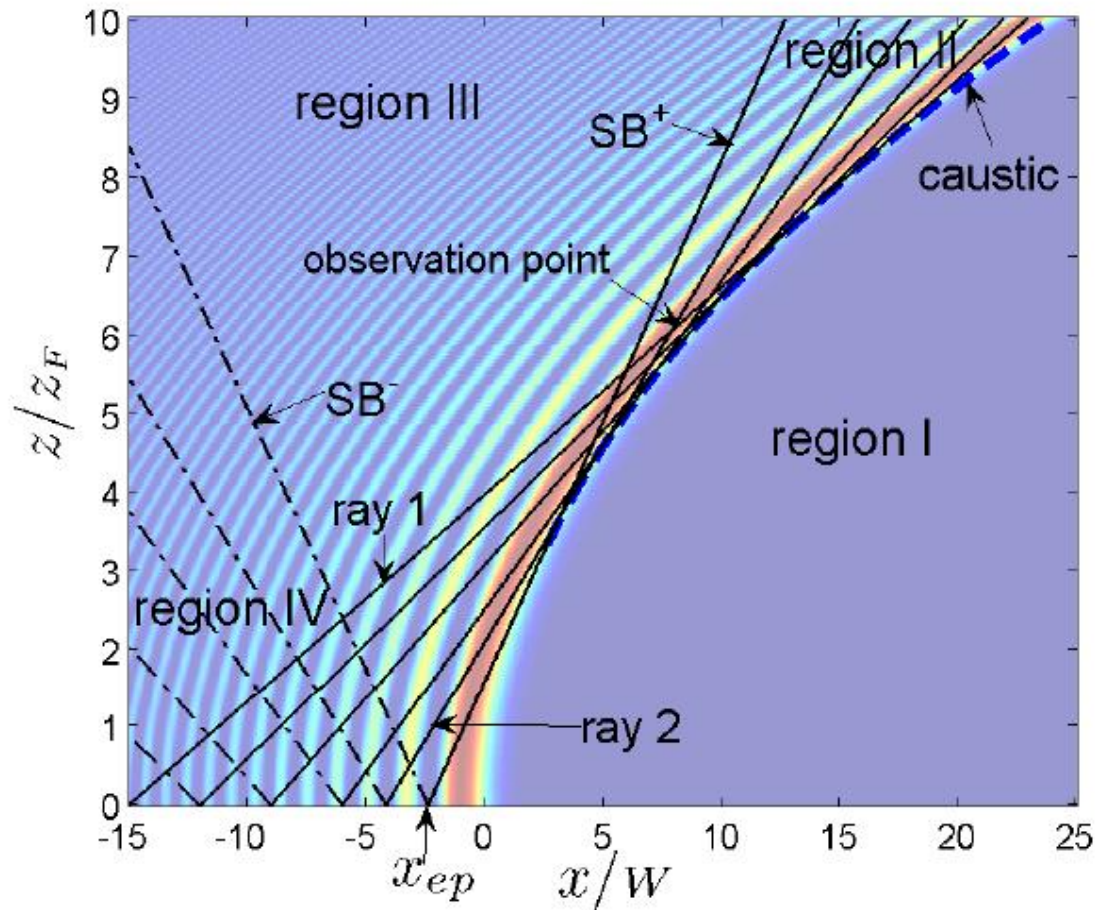
高斯光束

附加相位调制

实验



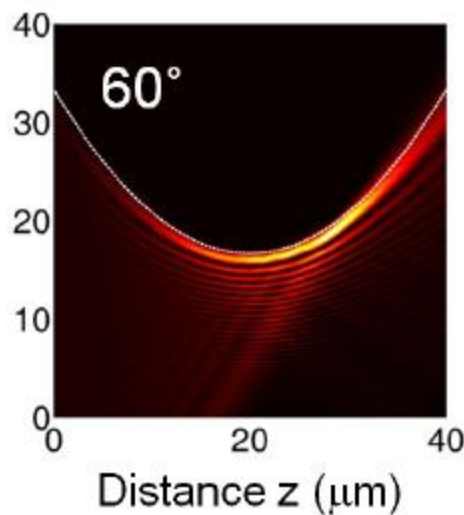
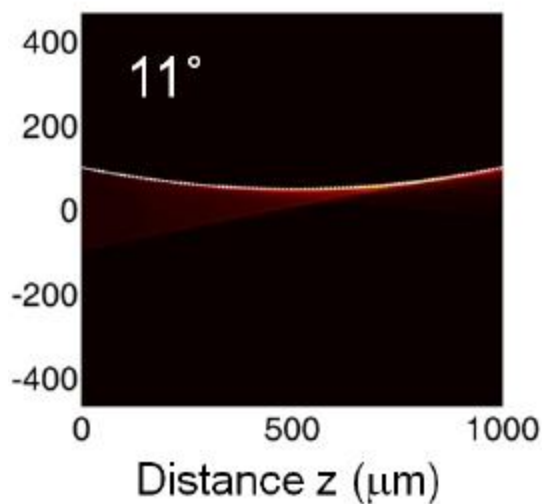
Geometric optics illustration



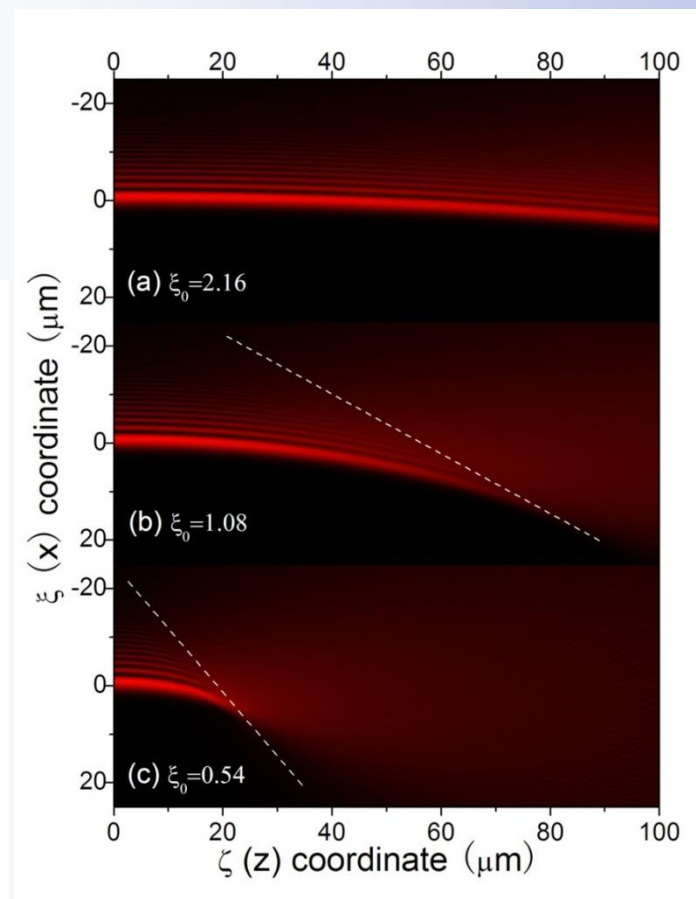
傍轴问题 (paraxial)

$$\psi(x) = -\frac{2}{3} \left(-\frac{\xi}{\xi_0} \right)^{3/2} - \frac{\pi}{4} - k \frac{\xi \sin \theta}{\cos(\theta - \theta_\xi)}$$

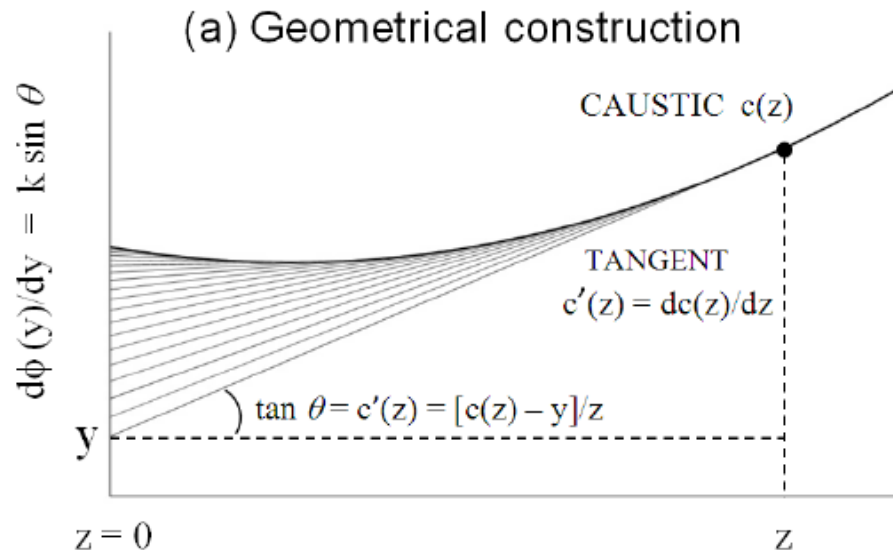
(b) Paraxial and non-paraxial caustics



?



Arbitrary caustic beam



Q3: 是否存在 nonparaxial nondiffracting beam?

Paraxial approximation: $\tan \theta \approx \sin \theta \approx \theta$

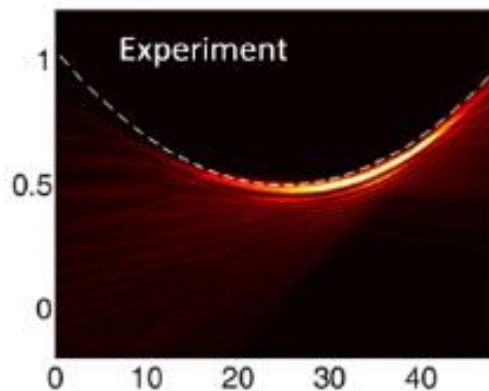
Nonparaxial $\neq$ nondiffracting

Airy beam

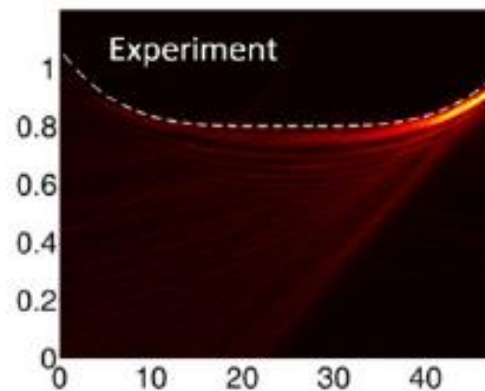
Table 1. In the Paraxial Approximation, this Table gives the Calculated Phase for Desired Acceleration Profiles as Shown.

Acceleration profile	Applied phase
Parabolic: $c(z) = az^2$	$\phi(y) = -4/3 a^{1/2} k y^{3/2}$
Quartic: $c(z) = az^4$	$\phi(y) = -16/21 (3a)^{1/4} k y^{7/4}$
Logarithmic: $c(z) = a \ln(bz)$	$\phi(y) = e^{-1} a^2 b k (1 - \exp[-y/a])$
Polynomial: $c(z) = az^n$ (for n even)	$\phi(y) = kn^2 y^2 \frac{[a(1-n)/y]^{1/n}}{(2n-1)(1-n)}$

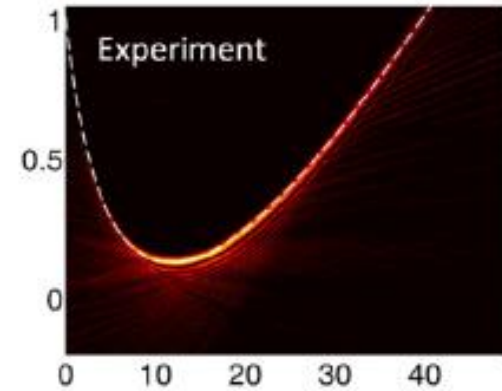
(a) Parabolic Beam



(b) Quartic Beam



(c) Logarithmic Beam





Nonparaxial [?] = nondiffracting

数学上：Airy 函数解是Helmholtz方程傍轴近似下的严格解。

实验上：当caustic beam角度变大后，beam明显偏离原来的profile。

Q3：是否存在 nonparaxial nondiffracting beam?



(III) Mathieu beam and Weber beam

- 回到最原始波动方程

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0$$

- 考虑场在y方向均匀，则变为二维Helmholtz方程

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0$$

- 分离变量令 $\phi = U(r) \exp(i\alpha\theta - i\omega t)$

则径向函数满足

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U}{\partial r} \right) + \left(k^2 - \frac{\alpha^2}{r^2} \right) U = 0$$

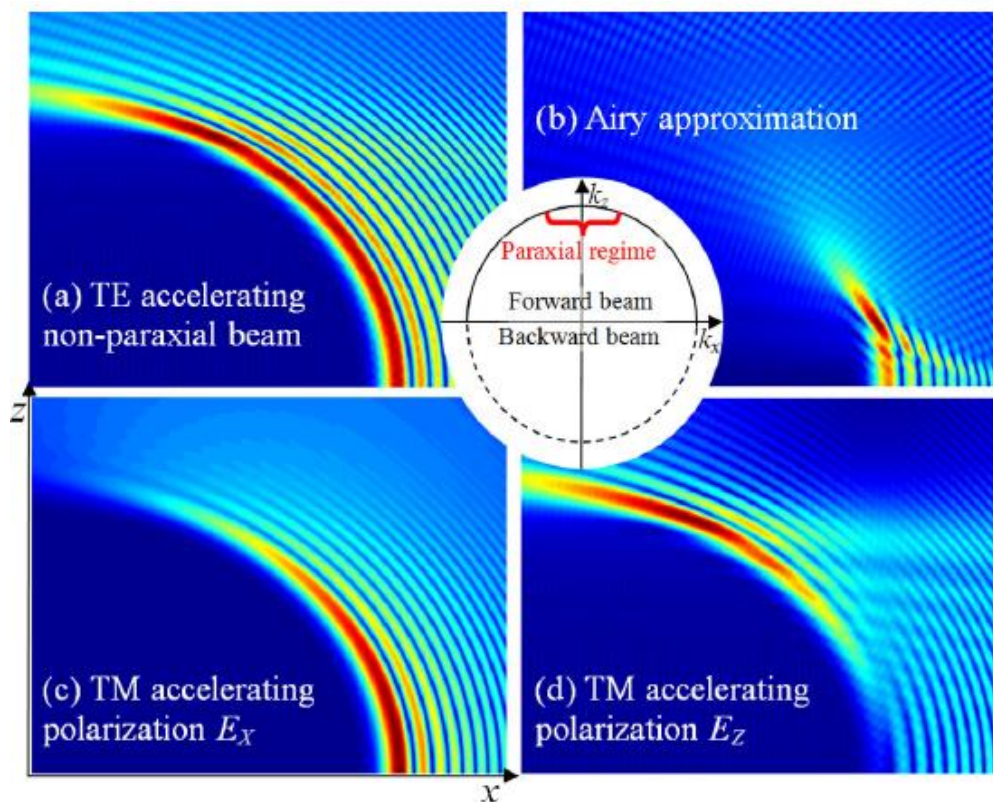
标准的Bessel方程

□ 如此得到径向不衍射解

$$U(r) = J_\alpha(kr)$$

$$\phi = J_\alpha(kr) \exp(i\alpha\theta - i\omega t)$$

注意： α 表示方位角向 θ 的行波波数，在求Bessel光束时候我们用零皮沿着 z 轴传播 $k_z \neq 0$ 。
 $(k_z = 0)$ ，从而 $\alpha \neq 0$ ，得到高阶传播的**不衍射弯曲波**。



这里 $\alpha=100$ （高阶贝塞尔函数）
 这时beam弯曲接近90度，
 大大突破了paraxial的限制！

角向贝塞尔波束
A-bessel beam



- ▣ 刚才的结果是在柱坐标下完成，其实可以进一步推广到椭圆坐标、抛物坐标等任意曲线坐标系下：

椭圆坐标系

$$\begin{cases} x = h \sinh \xi \sin \eta \\ z = h \cosh \xi \cos \eta \end{cases}$$

$\xi \in [0, \infty)$ 对应于 “径向”
 $\eta \in [0, 2\pi)$ 对应于 “方位角”
 $h = \sqrt{|a^2 - b^2|}$

$$\frac{d^2 R(\xi)}{d\xi^2} - (\beta - 2q \cosh 2\xi) R(\xi) = 0$$

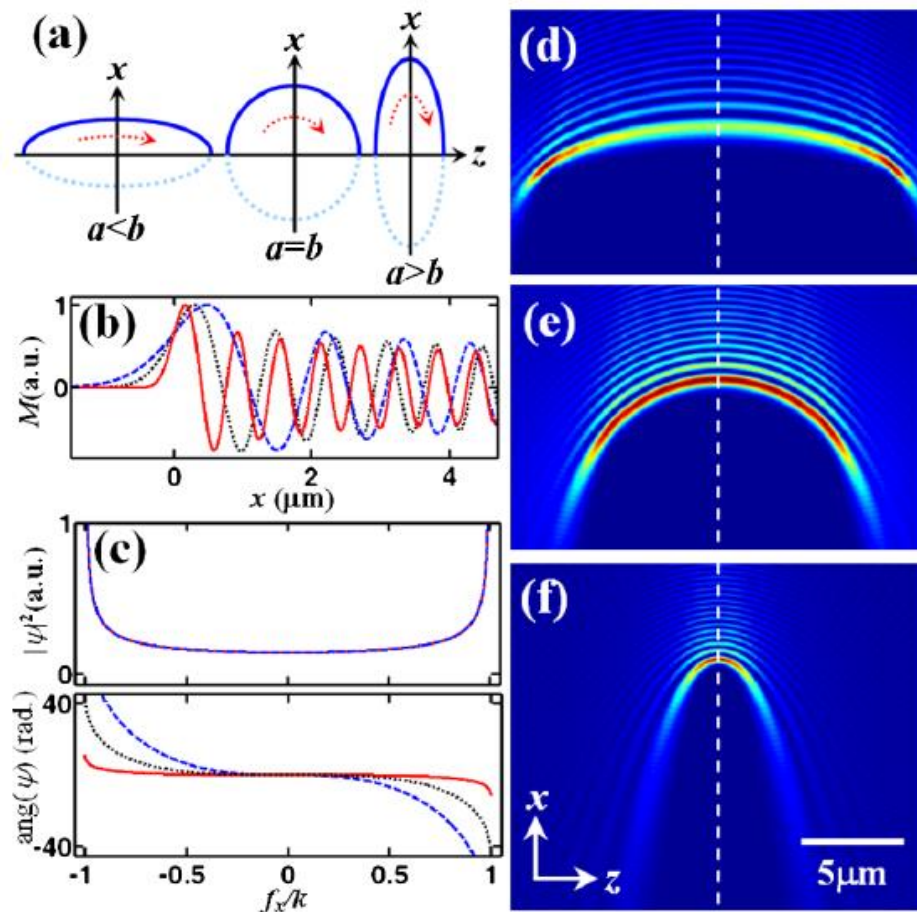
$$\frac{d^2 \Theta(\eta)}{d\eta^2} - (\beta - 2q \cosh 2\xi) \Theta(\eta) = 0$$

$$M(\xi, q) = R_m(\xi, q) (ce_m(\eta; q) - ise_m(\eta; q))$$

径向Mathieu函数

角向Mathieu函数

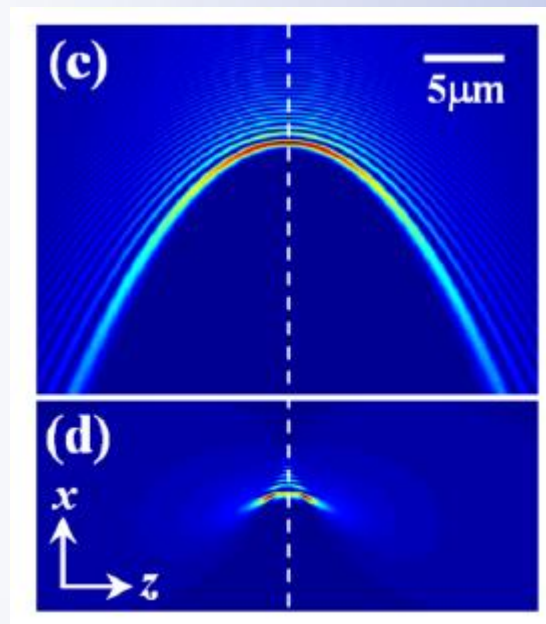
Mathieu beam



Parabolic coordinate

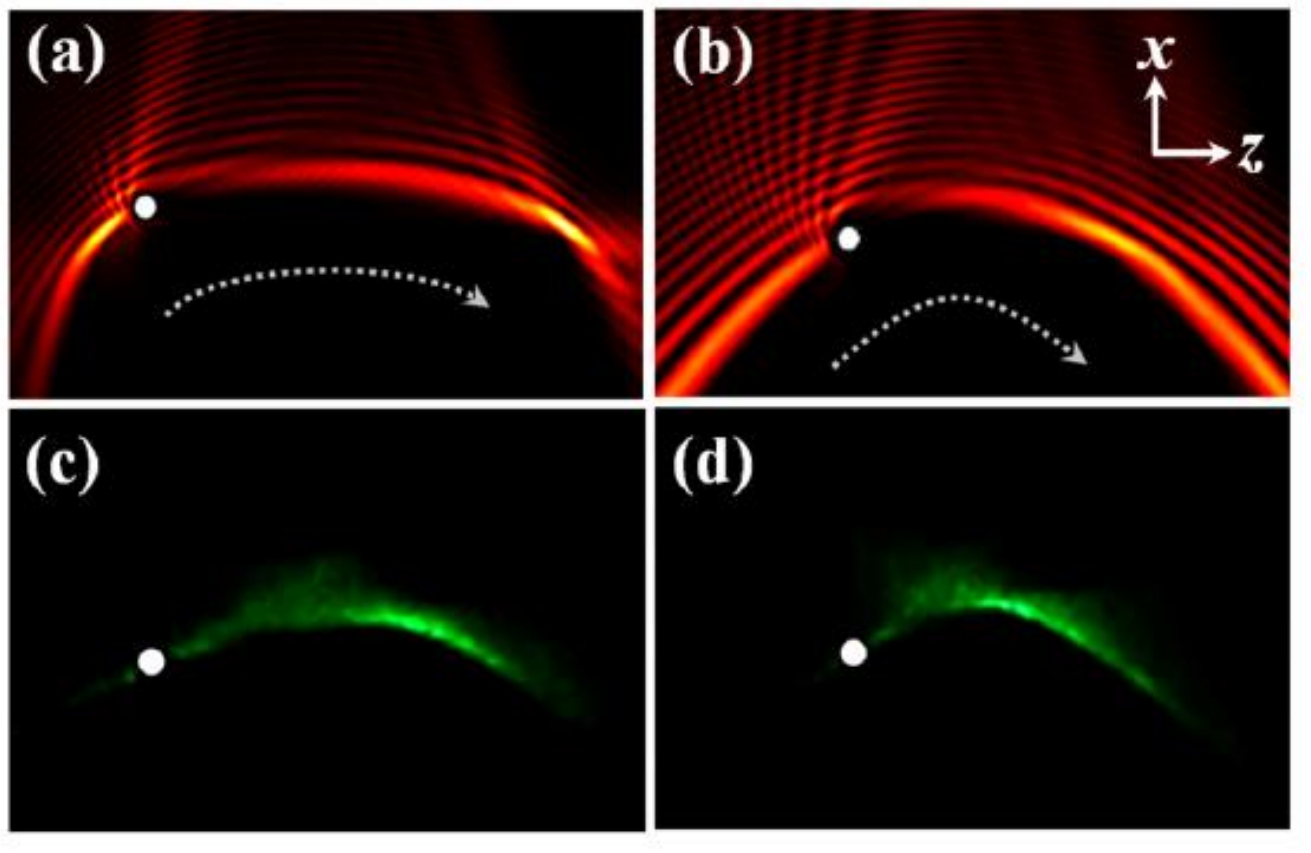
$$\frac{d^2\Phi(\sigma)}{d\sigma^2} + (k^2\sigma^2 + 2k\gamma)\Phi(\sigma) = 0,$$

$$\frac{d^2\vartheta(\tau)}{d\tau^2} + (k^2\tau^2 - 2k\gamma)\vartheta(\tau) = 0,$$



Weber beam

Self-healing property



Phys. Rev. Lett. 109, 193901 (2012)



Summarizing nondiffracting beams

$$\nabla^2 u + k^2 u = 0$$

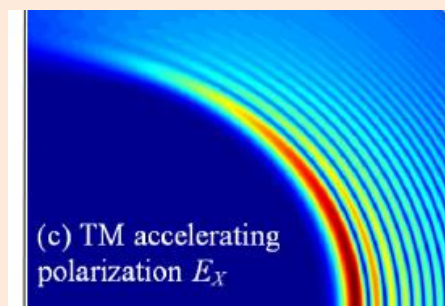
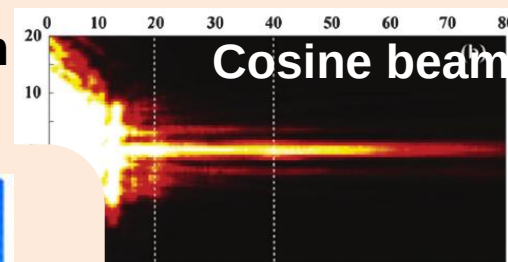
Cylindrical coordinate

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dR}{dr} \right) + \left(k^2 - \lambda - \frac{\mu}{r^2} \right) R = 0$$

$$R(r) = CJ_m(k_r r) + DN_m(k_r r)$$

$$\frac{d^2 Z}{dz^2} + \lambda Z = 0$$

$$\frac{d^2 \Phi}{d\theta^2} + \mu \Phi = 0$$



Elliptical coordinate

$$M(\xi, q) = R_m(\xi, q)(ce_m(\eta; q) - ise_m(\eta; q))$$

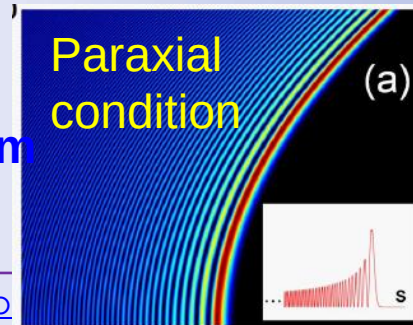
Mathieu beam

Parabolic coordinate

$$W(\eta, \xi; \gamma) = \frac{1}{2\pi} \begin{bmatrix} |\Gamma_1|^2 P_0(\sqrt{2k}\xi; \gamma) P_0(\sqrt{2k}\eta; -\gamma) \\ + 2i |\Gamma_3|^2 P_1(\sqrt{2k}\xi; \gamma) P_1(\sqrt{2k}\eta; -\gamma) \end{bmatrix}$$

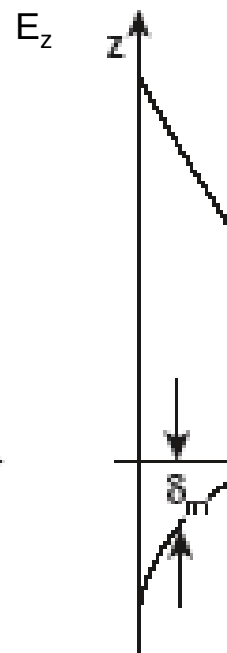
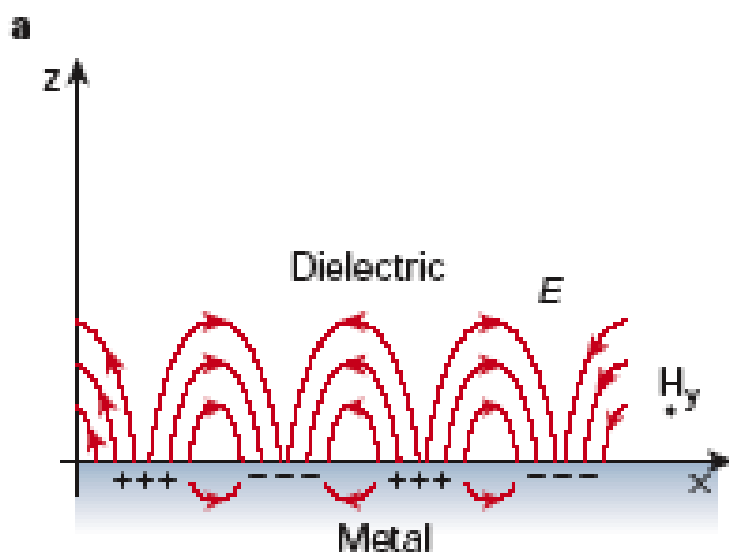
Airy beam

Weber beam



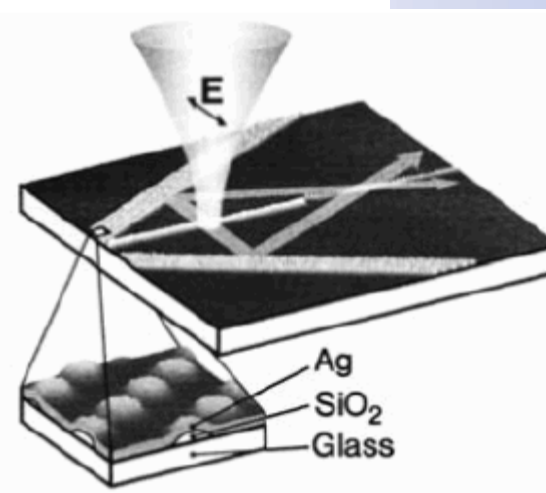
(IV) Plasmon counterparts

Two dimensional nature of SPP



$$\delta_d \sim 100 \text{ nm}$$

$$\delta_m \sim 10 \text{ nm}$$





Airy plasmon: a nondiffracting surface wave

Alessandro Salandrino* and Demetrios N. Christodoulides

CREOL/College of Optics and Photonics, University of Central Florida, Orlando, Florida 32816, USA

Received March 1, 2010; accepted May 17, 2010;

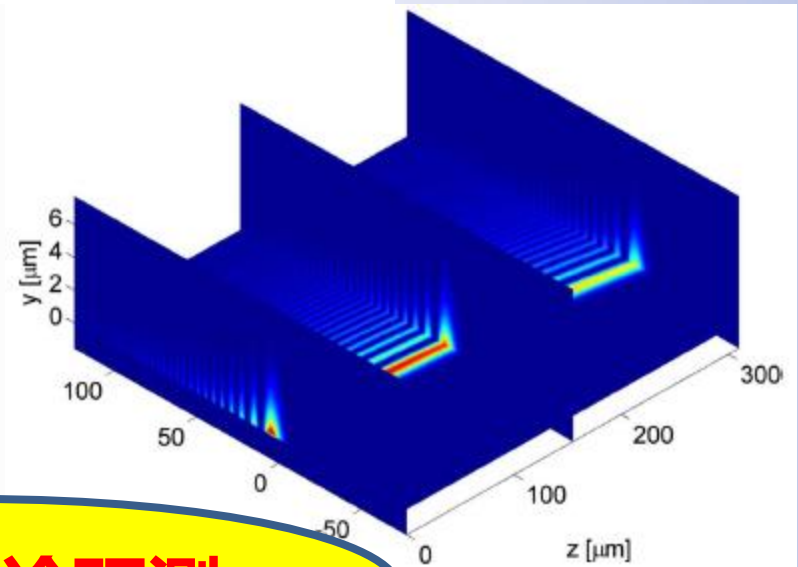
posted June 2, 2010 (Doc. ID 124913); published June 11, 2010

针对SPP的TM场的方程

$$\nabla^2 E_{dy} + k_0^2 \varepsilon_d E_{dy} = 0. \quad \frac{\partial^2 A}{\partial x^2} + 2ik_z \frac{\partial A}{\partial z} = 0.$$

很容易也可得到Airy函数的解

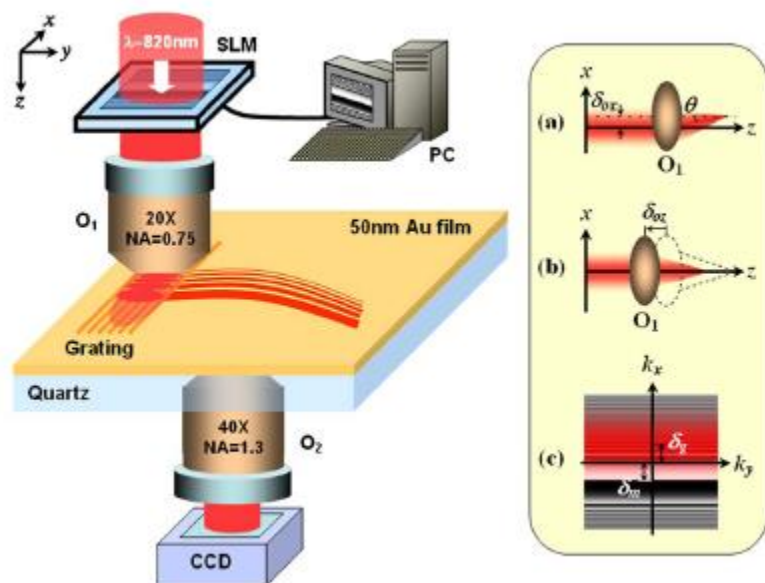
$$\begin{aligned} A(x, z) = & Ai \left[\frac{x}{x_0} - \left(\frac{z}{2k_z x_0^2} \right)^2 + i \frac{az}{k_z x_0^2} \right] \\ & \times \exp \left[i \left(\frac{x + a^2 x_0}{2x_0} \frac{z}{k_z x_0^2} - \frac{1}{12} \left(\frac{z}{k_z x_0^2} \right)^3 \right) \right] \\ & \times \exp \left[a \frac{x}{x_0} - \frac{a}{2} \left(\frac{z}{k_z x_0^2} \right)^2 \right]. \end{aligned}$$



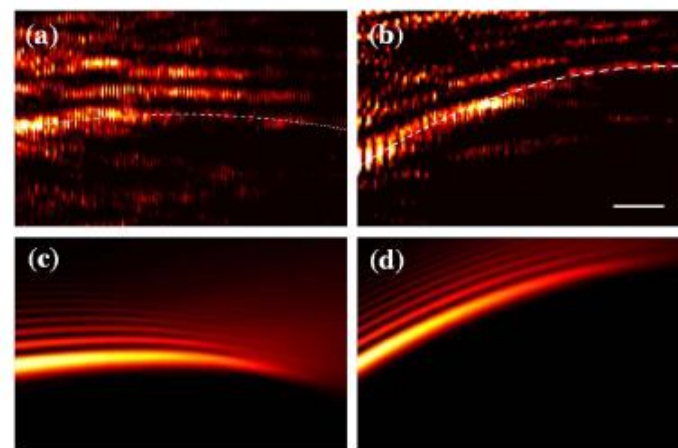
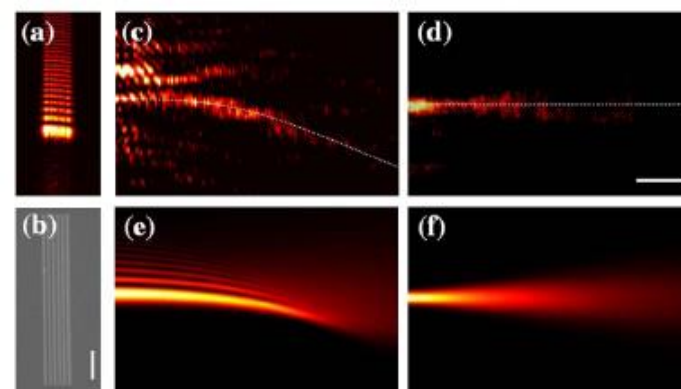
Only理论预测

Plasmonic Airy beams with dynamically controlled trajectories

Peng Zhang,^{1,2,†} Sheng Wang,^{1,†} Yongmin Liu,^{1,†} Xiaobo Yin,^{1,3} Changgui Lu,¹ Zhigang Chen,² and Xiang Zhang^{1,3,*}



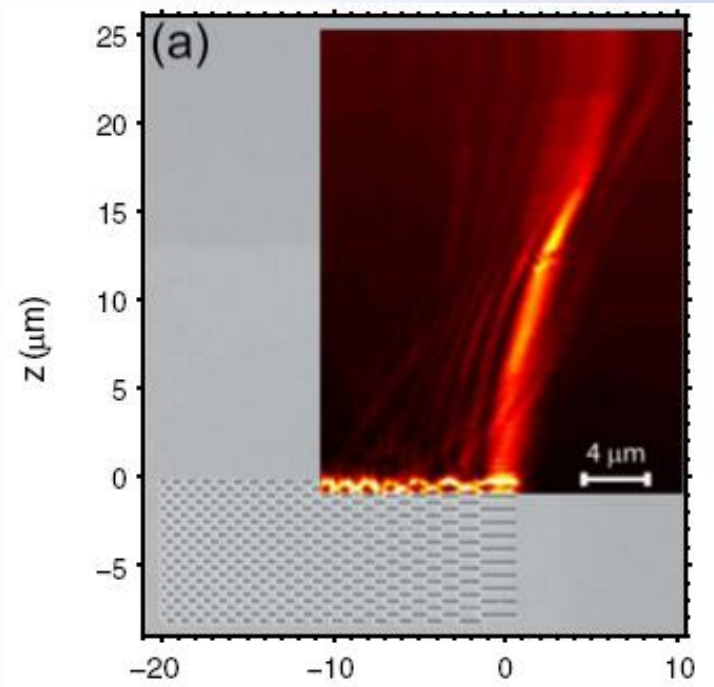
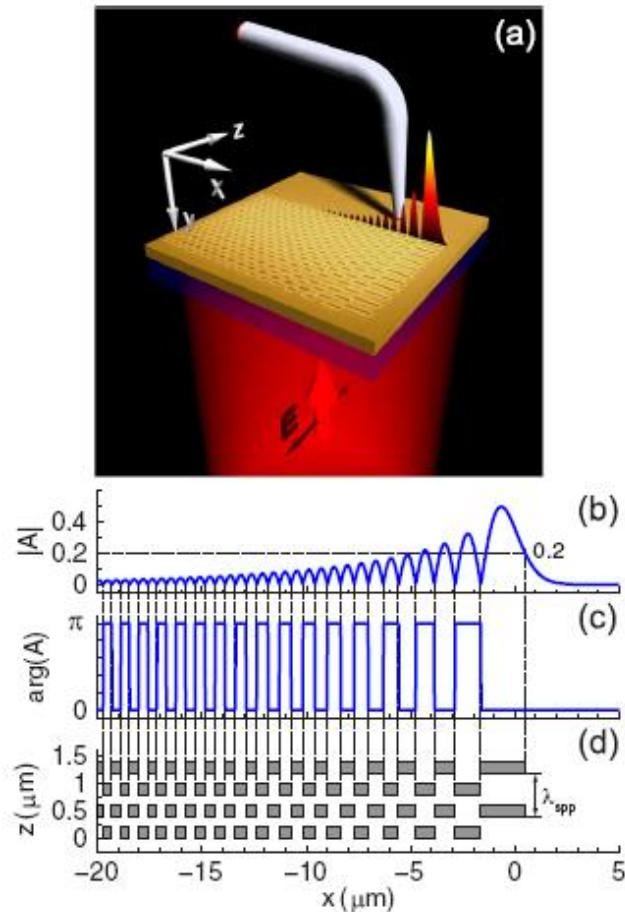
Coupling a well generated Airy beam to SPP





Generation and Near-Field Imaging of Airy Surface Plasmons

Alexander Minovich,¹ Angela E. Klein,² Norik Janunts,² Thomas Pertsch,² Dragomir N. Neshev,¹ and Yuri S. Kivshar¹

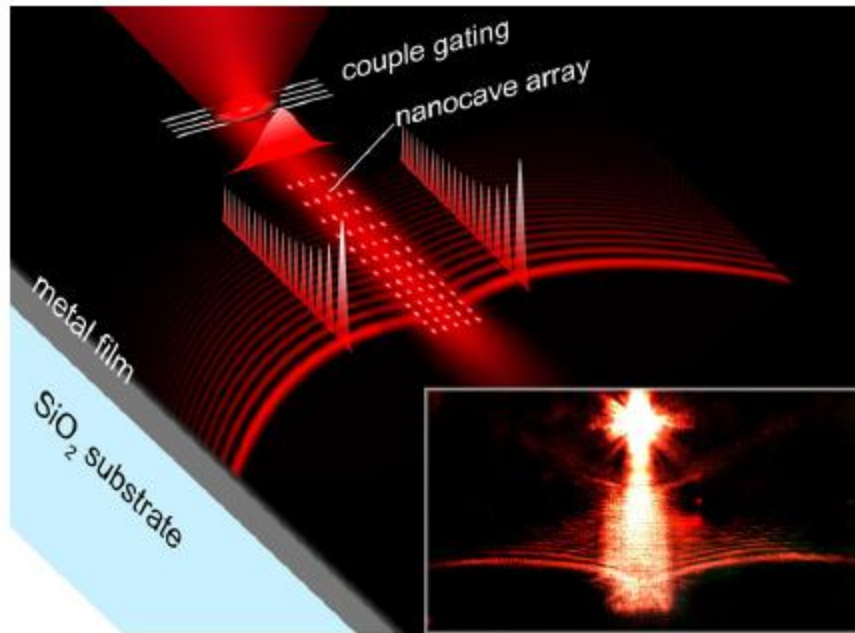


**Using well designed nano-grating
to couple Airy plasmon
(coupling process)**

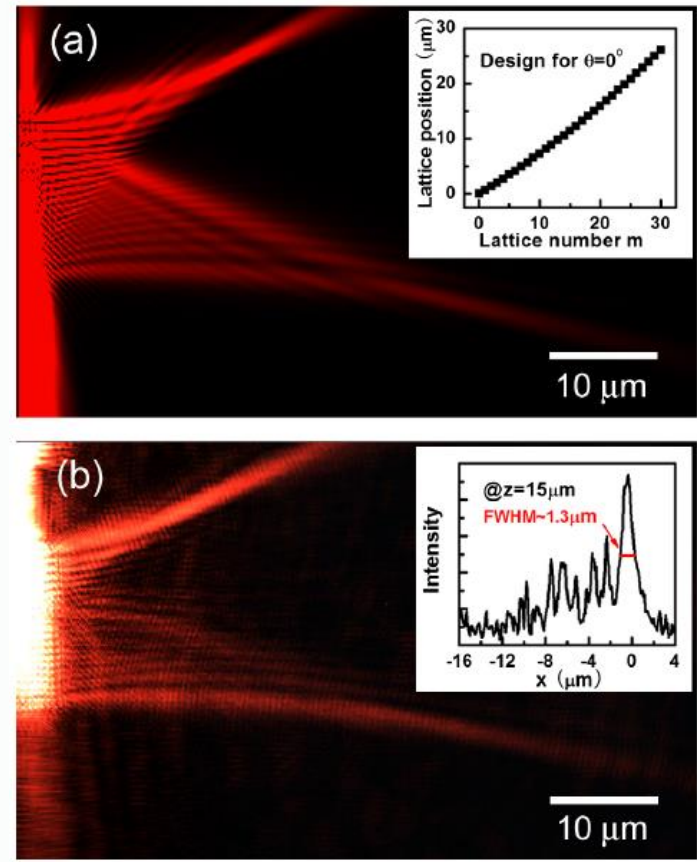


Plasmonic Airy Beam Generated by In-Plane Diffraction

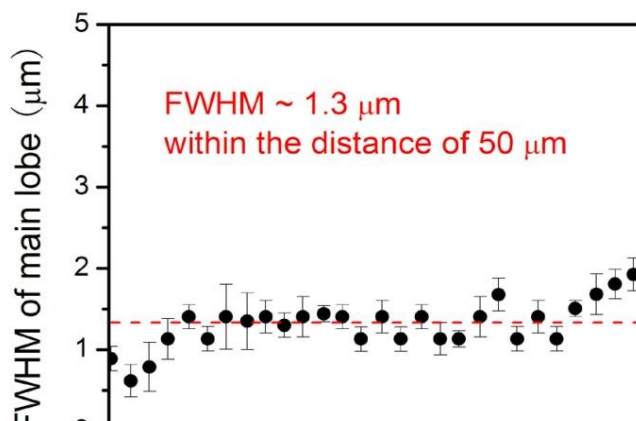
L. Li, T. Li,* S. M. Wang, C. Zhang, and S. N. Zhu



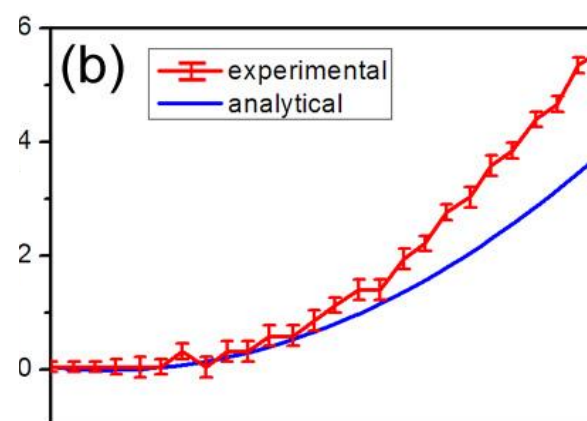
**Generating Airy plasmon totally
on planar dimension.
(independent to coupling process)**



Non-spreading beam



Parabolic trajectory

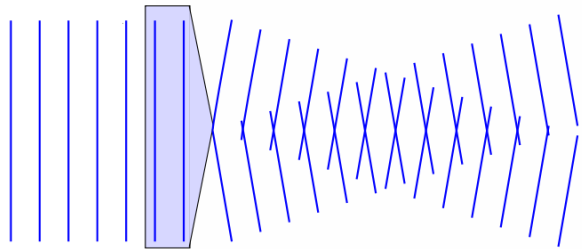


Q4: 是否存在 straight nondiffracting SPP?



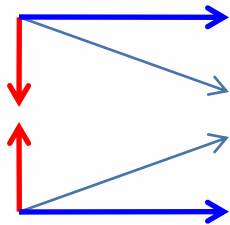
Self-healing property

Cosine-Gauss plasmon

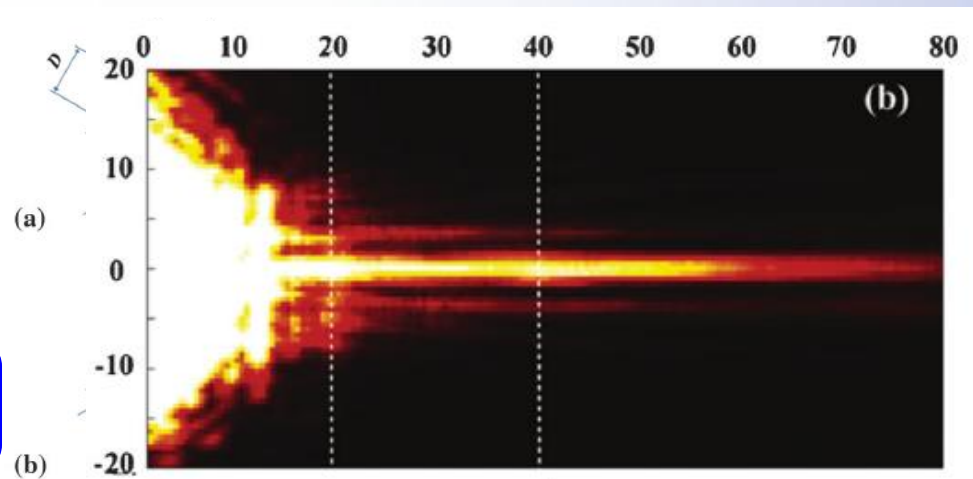


Bessel beam是柱坐标下平面波的叠加
而SPP仅仅存在金属表面2D体系，不
可能适用柱坐标！

等效过程



$$\exp(ik_y y) + \exp(-ik_y y) = \cos(k_y y)$$



PRL **109**, 093904 (2012)

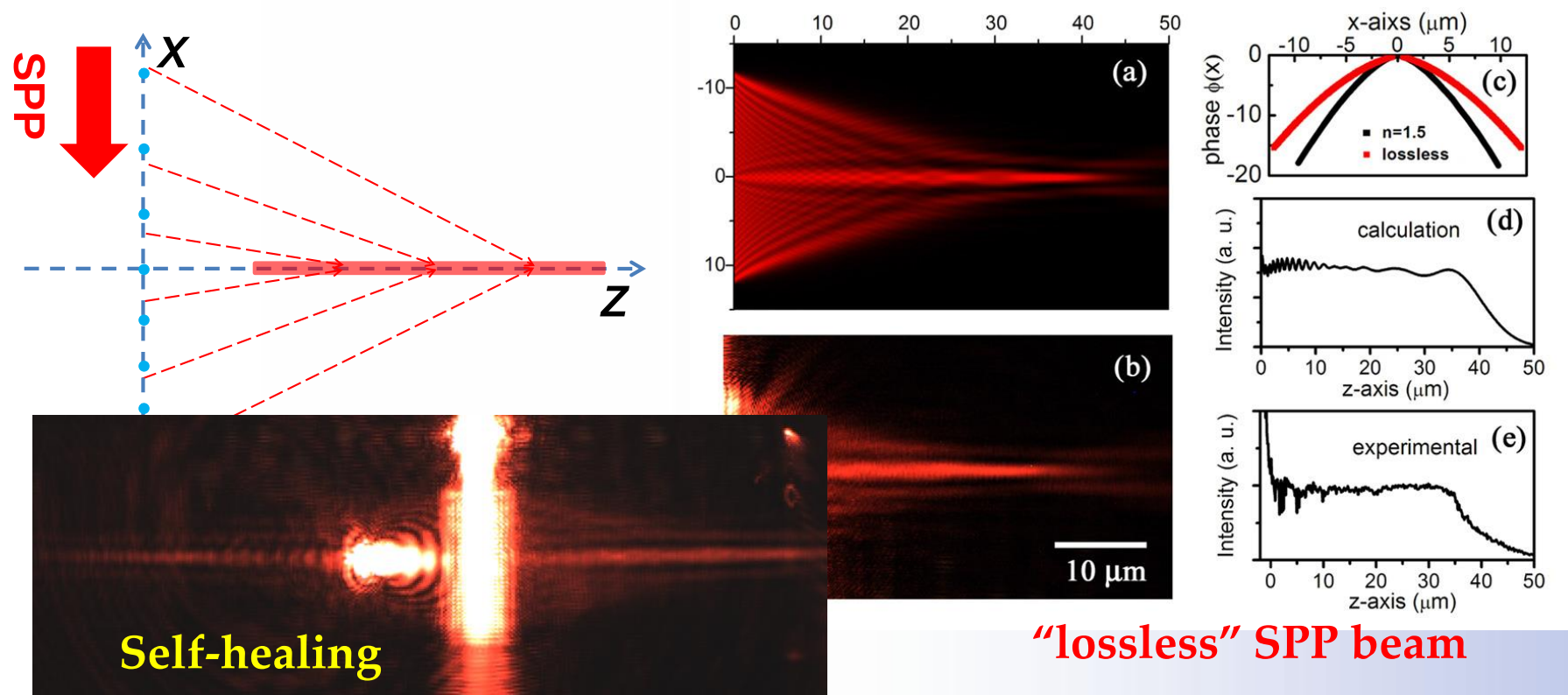
PHYSICAL REVIEW LETTERS

week ending
31 AUGUST 2012

Cosine-Gauss Plasmon Beam: A Localized Long-Range Nondiffracting Surface Wave

Jiao Lin,^{1,2} Jean Dellinger,³ Patrice Genevet,^{1,4} Benoit Cluzel,³ Frederique de Fornel,³ and Federico Capasso^{1,*}

Our strategy- diffraction process



PRL 110, 046807 (2013)

PHYSICAL REVIEW LETTERS

week ending
25 JANUARY 2013

Collimated Plasmon Beam: Nondiffracting versus Linearly Focused

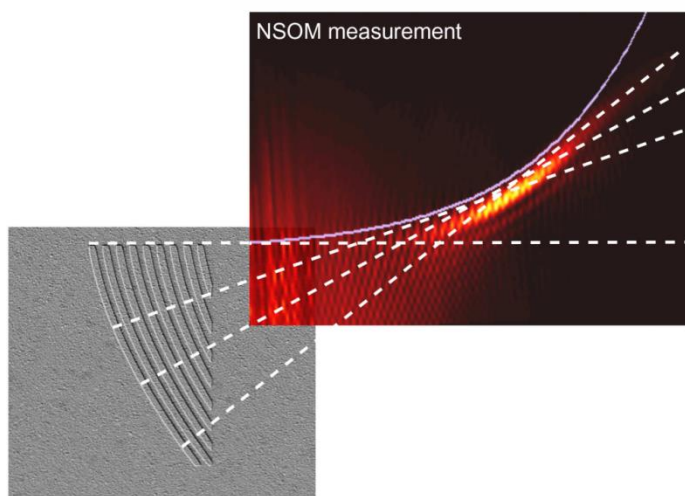
L. Li, T. Li,* S. M. Wang, and S. N. Zhu

Arbitrary bending

How about nonparaxial SPP?

Caustic方法获得任意曲线传播

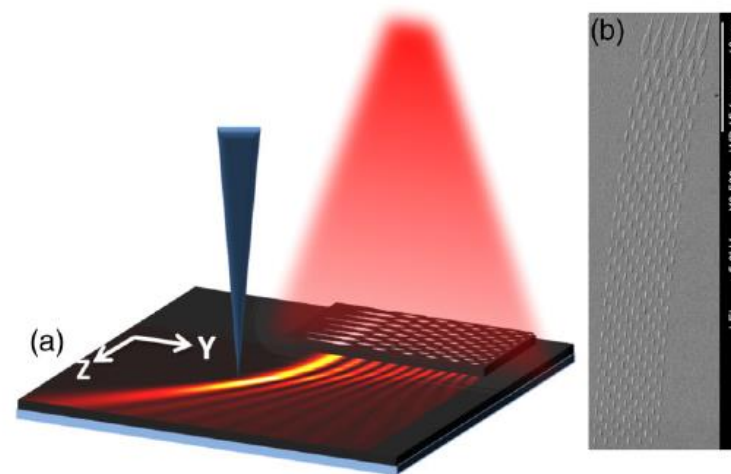
Mathieu & Weber SPP



PRL 112, 023903 (2014)

PRL 113, 123902 (2014)

PHYSICAL



PRL 113, 123902 (2014)

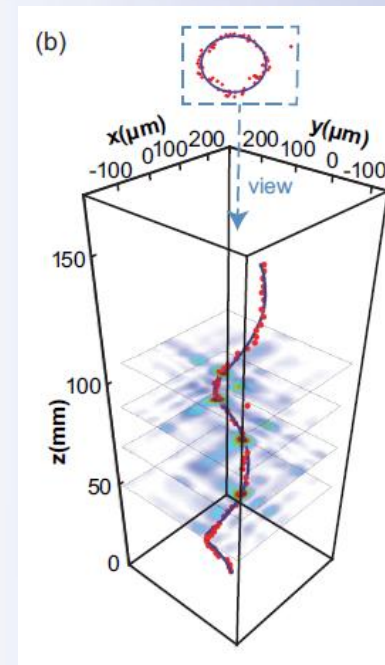
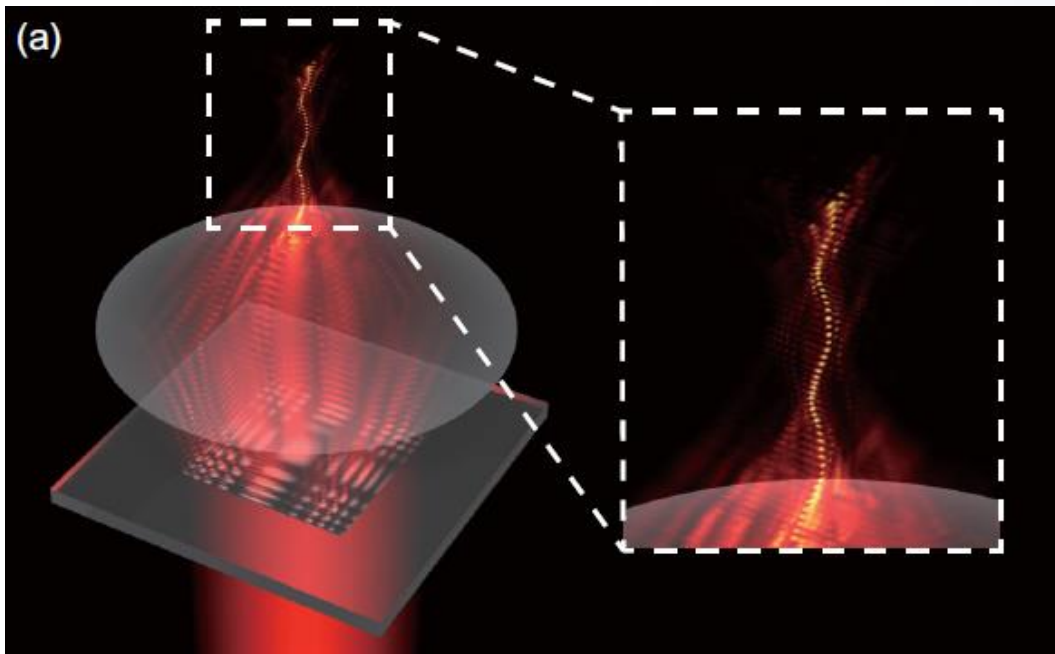
Spatial winding beam

PHYSICAL REVIEW A **94**, 013829 (2016)

Modulation in Fourier space

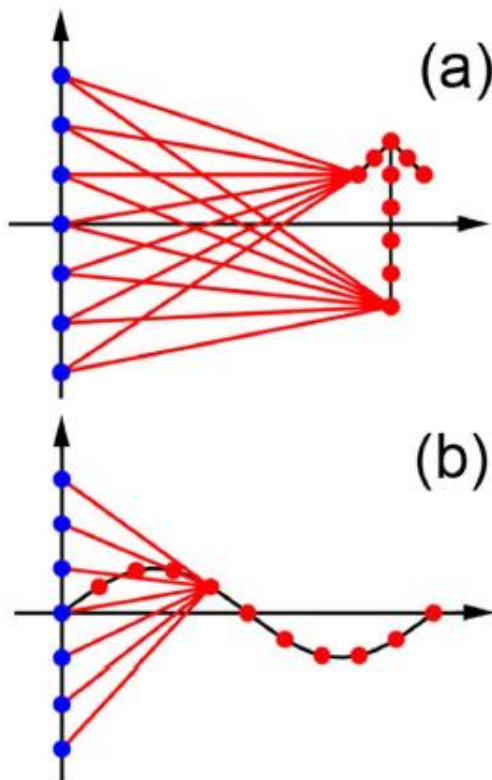
Winding light beams along elliptical helical trajectories

Yuanhui Wen,¹ Yujie Chen,^{1,*} Yanfeng Zhang,¹ Hui Chen,¹ and Siyuan Yu^{1,2,†}

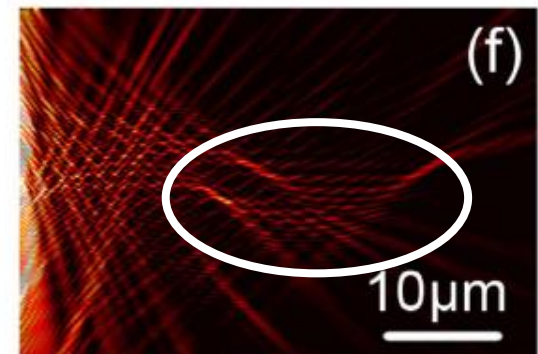
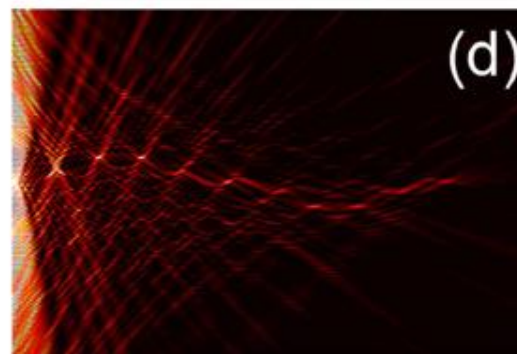
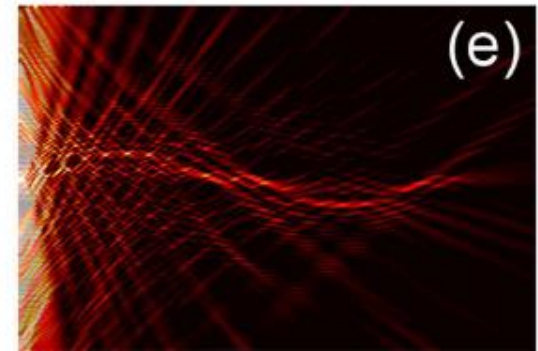
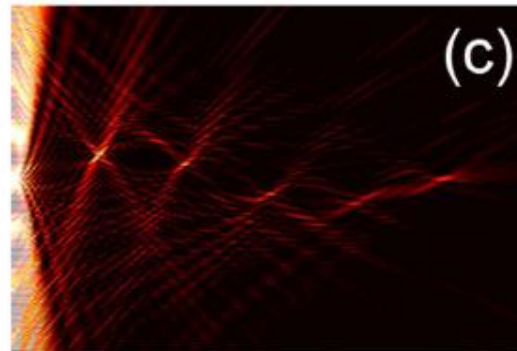


Our progress

Plasmonic **hologram** for **indefinite beam**

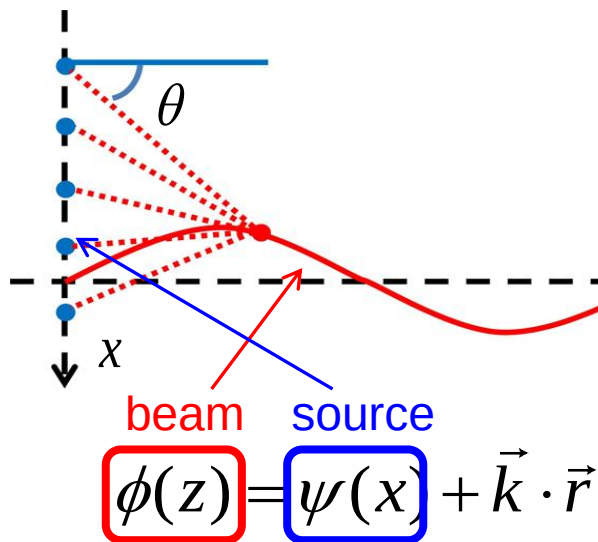


Sine-function oscillating beam



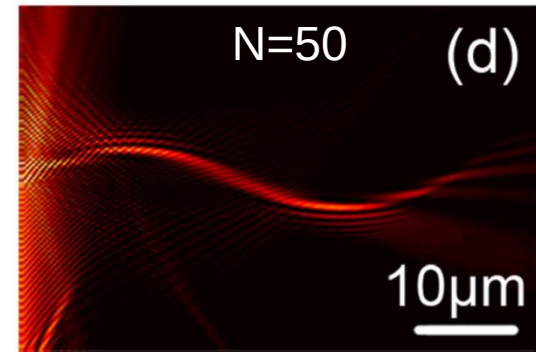
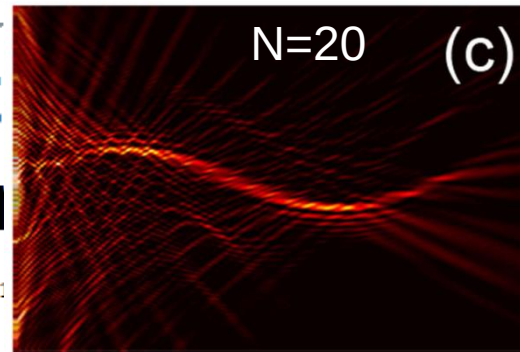
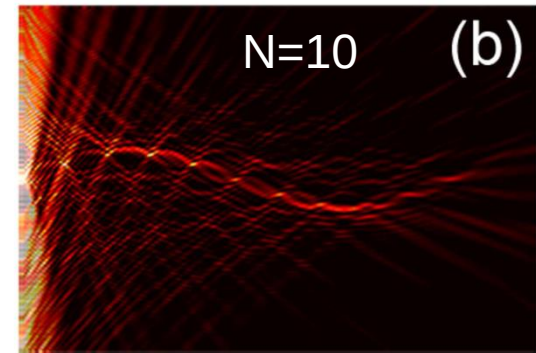
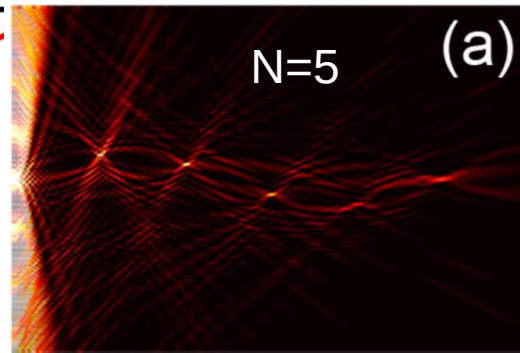
Beam collapses

Indefinite beam holography



$$E^{source}(x, z=0) = \sum_n \frac{1}{\sqrt{r_n}} \exp[i(\phi_n(z) + \vec{k} \cdot \vec{r}_n)]$$

$$E^{source}(x, z=0) = A(x) \times \exp[i\psi^H(x)]$$

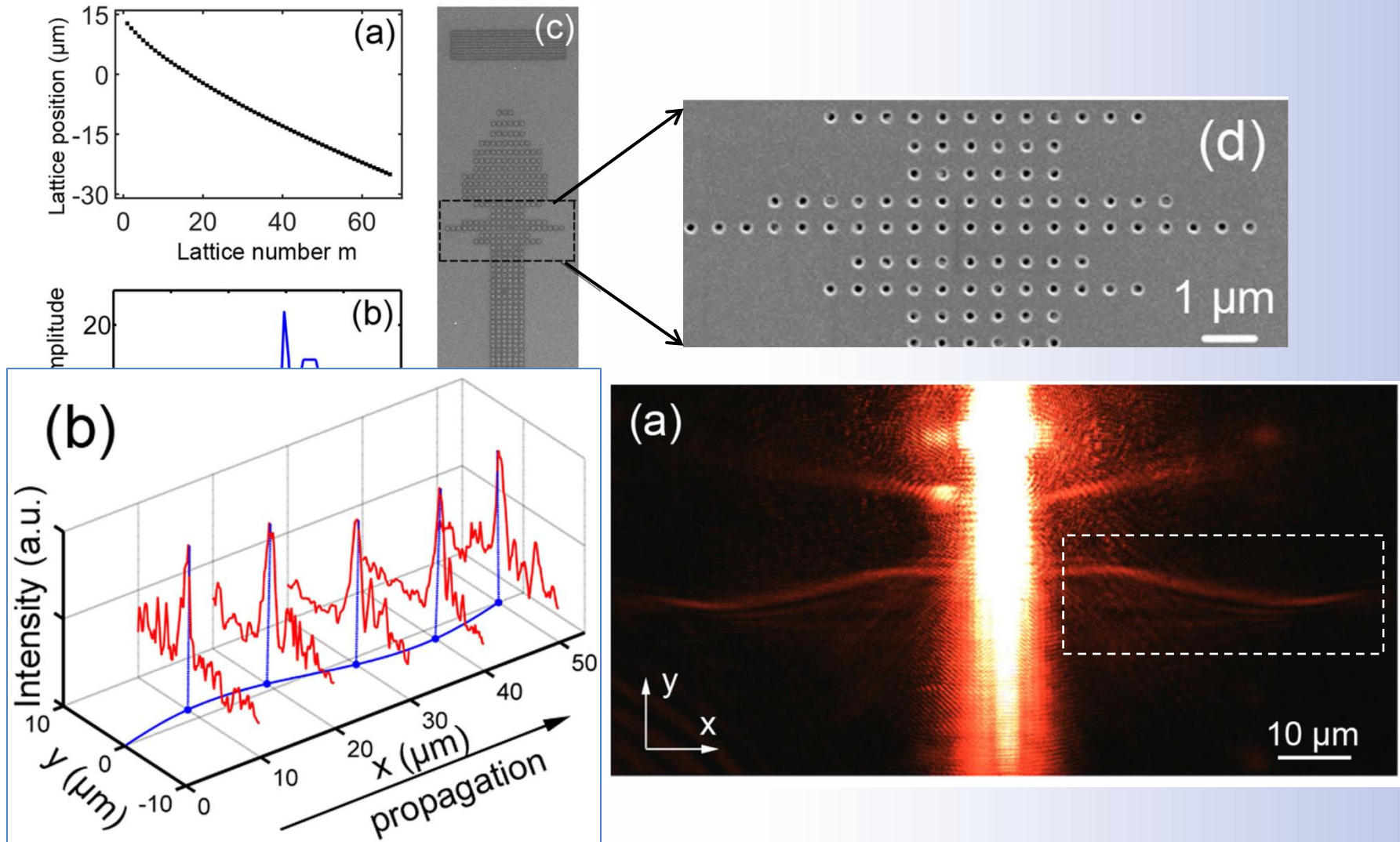


OPEN

Indefinite
Engineer

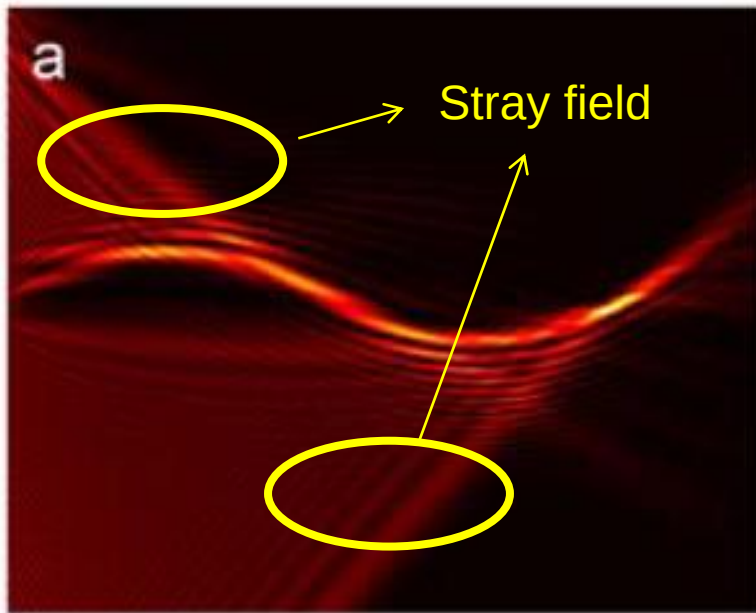
J. Chen^{1,2}, L. Li^{1,2}, T. Li

Experimental result

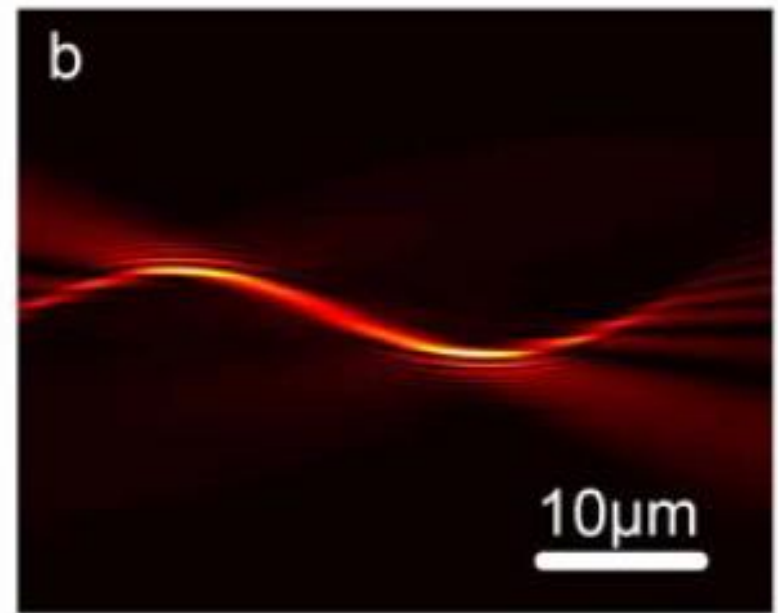


Comparison with caustic beam

Theoretical results



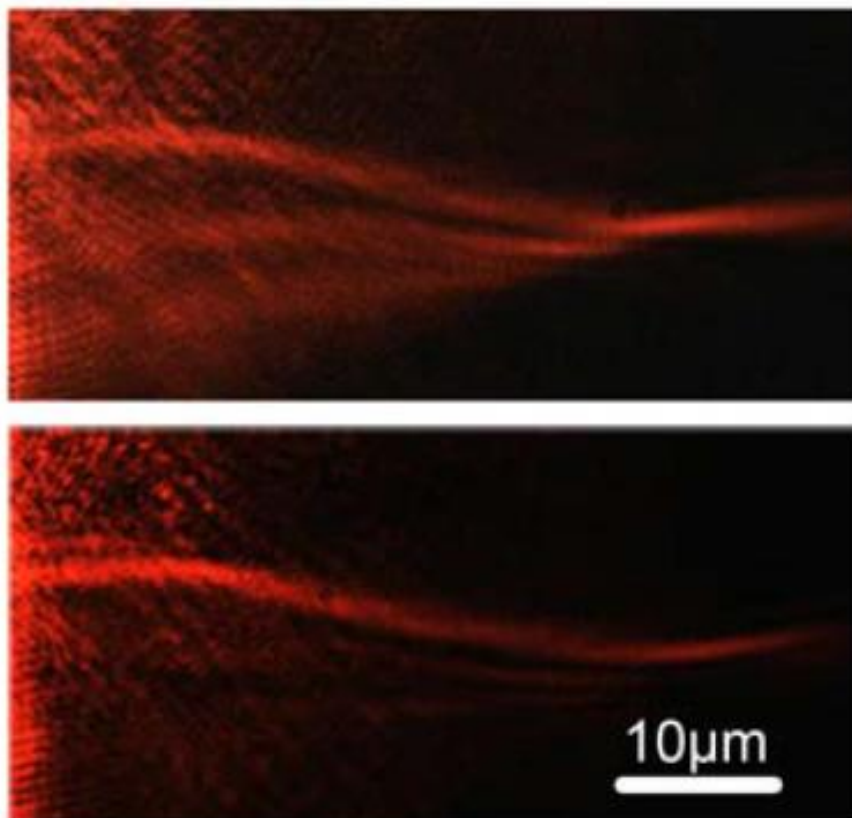
Caustic beam design



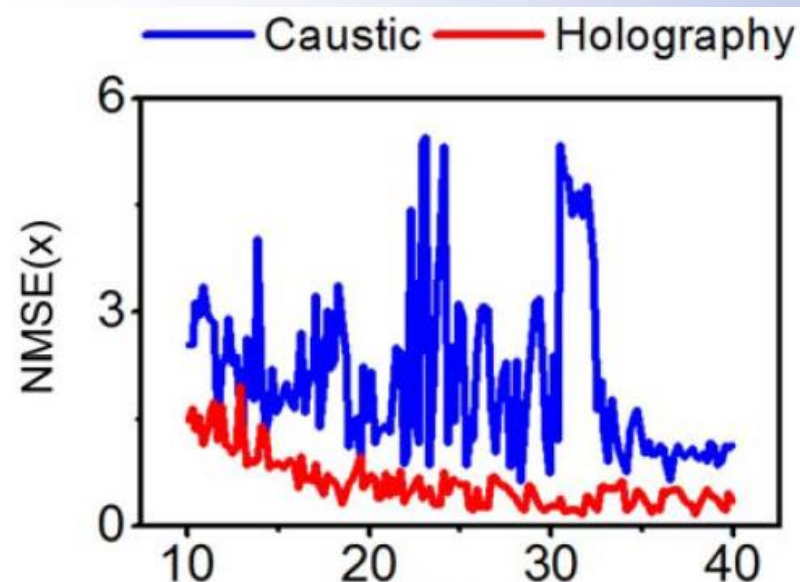
Holographic design

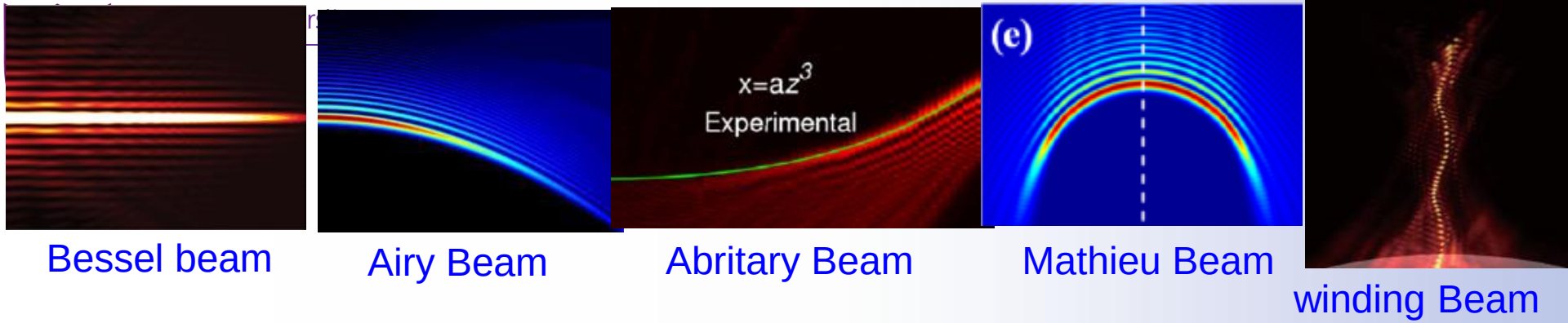
Comparison with caustic beam

Experiments

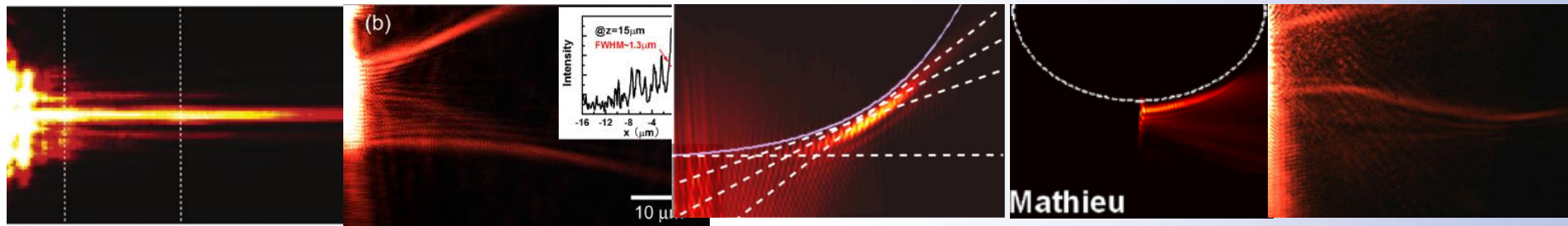


$$NMSE(x) = \frac{\sum |I'(y) - I(y)|^2}{\sum |I(y)|^2}$$





Nondiffracting beams	Bessel beam	Airy beam	Nonparaxial beam	Mathieu & Webber	Winding beam
Free space	1987	2007	2011	2012	2016
SPP	2012	2011	2014	2014	2016



Cosine-Gauss Plasmon Airy Plasmon Arbitrary SPP Mathieu SPP Sine SPP

More opportunities and challenges are in applications!



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Phys. Rev. Lett. 113, 123902 (2014)

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Acknowledgement

